Lines of Sight: Activities Related to Visual Perspective

Anna Davis

Ohio Dominican University

MathFest 2024

Two Activities:

1. Where was Eye?
2. Shady Business

Both can be done together in a two-hour math circle.

Questions?

davisa@ohiodominican.edu

Lines of Sight

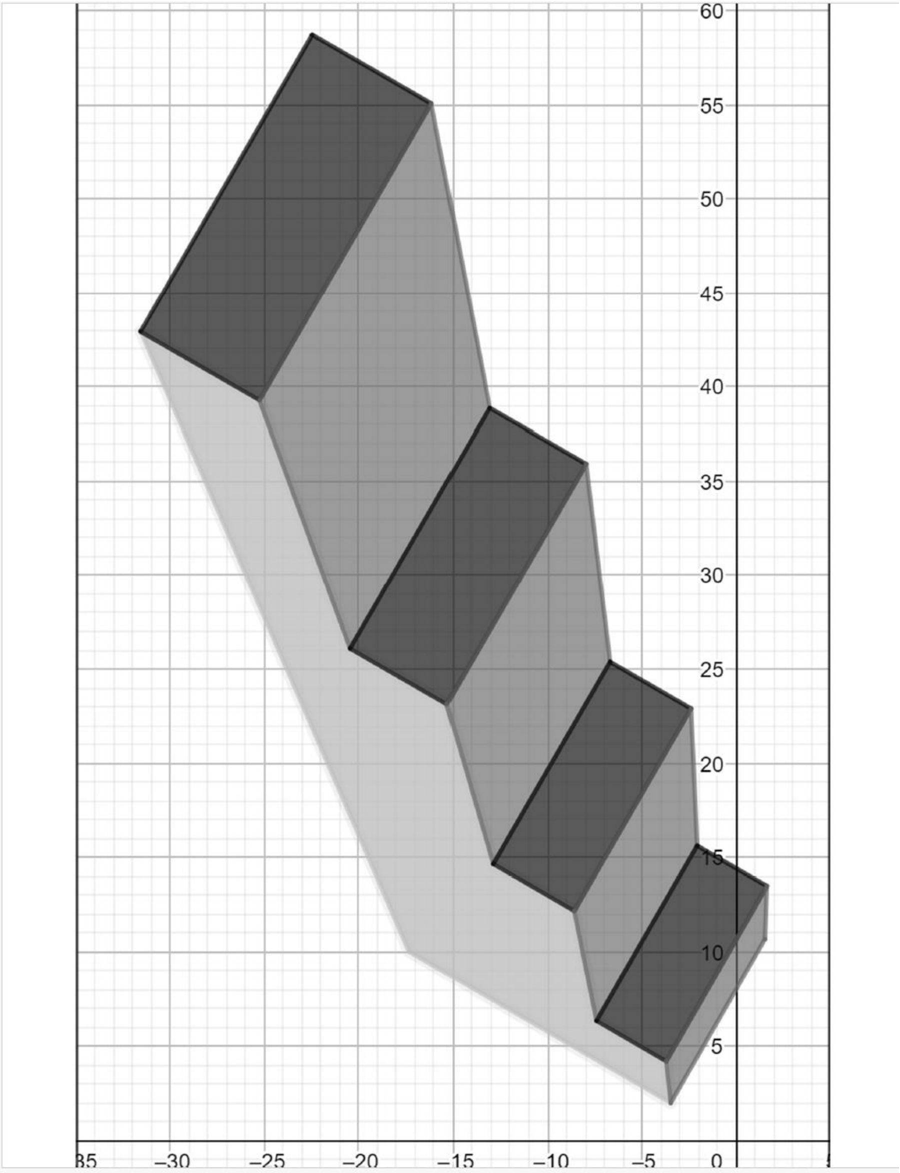
XIMERA

XIMERA is a free and open-source platform offering tools for authoring and publishing interactive educational content, including textbooks, assessments, and online courses.

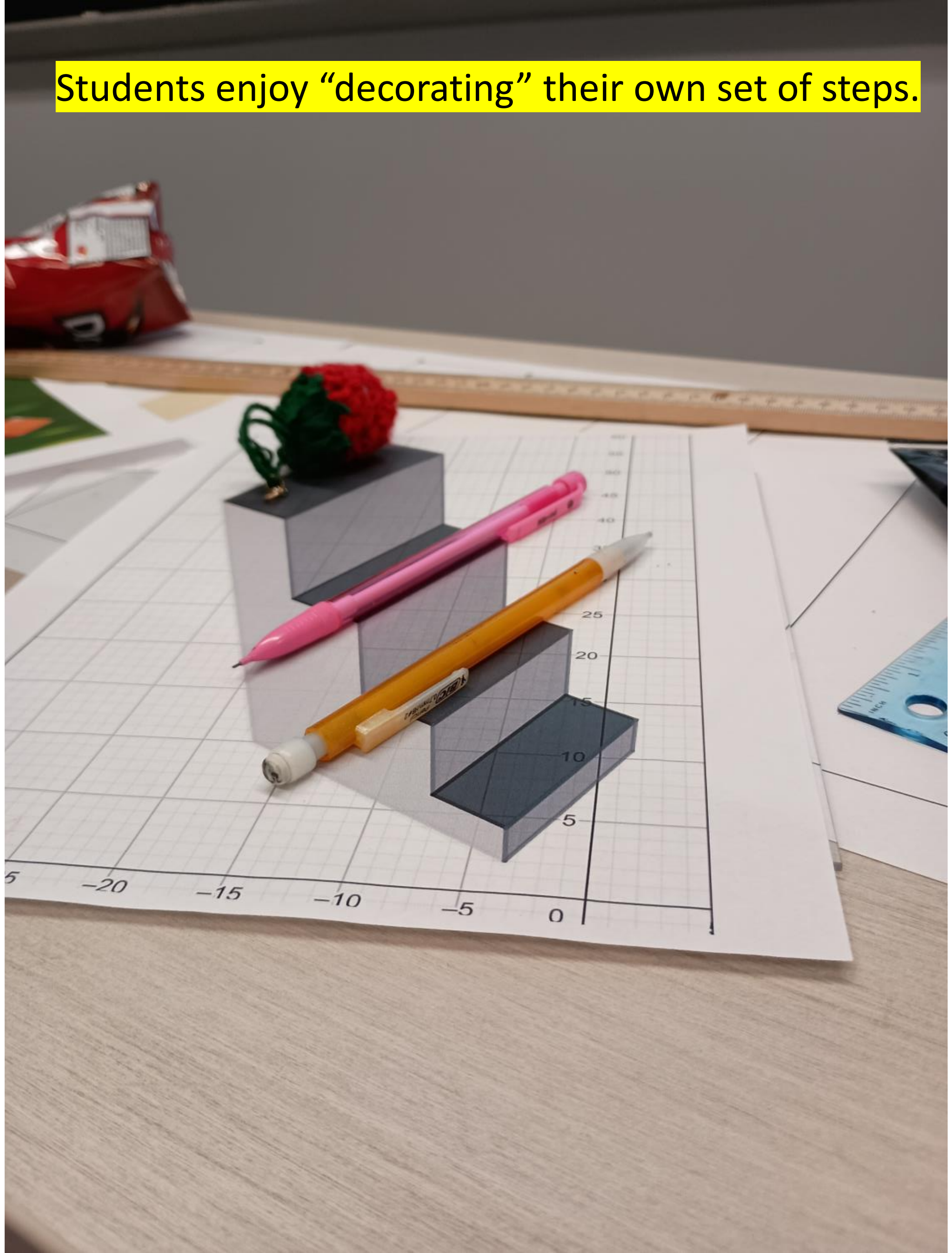
Exhibitor Booth 310

Shady Business


www.ofir-eruim.co.il
Ana Kogan



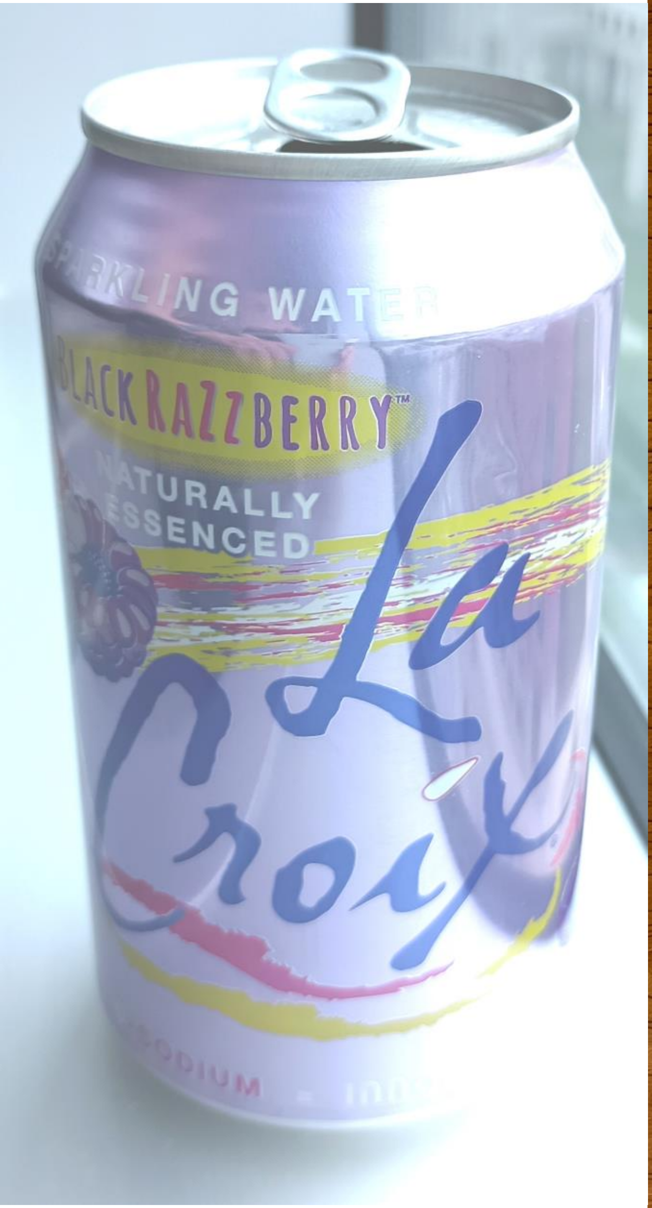
Students enjoy “decorating” their own set of steps.

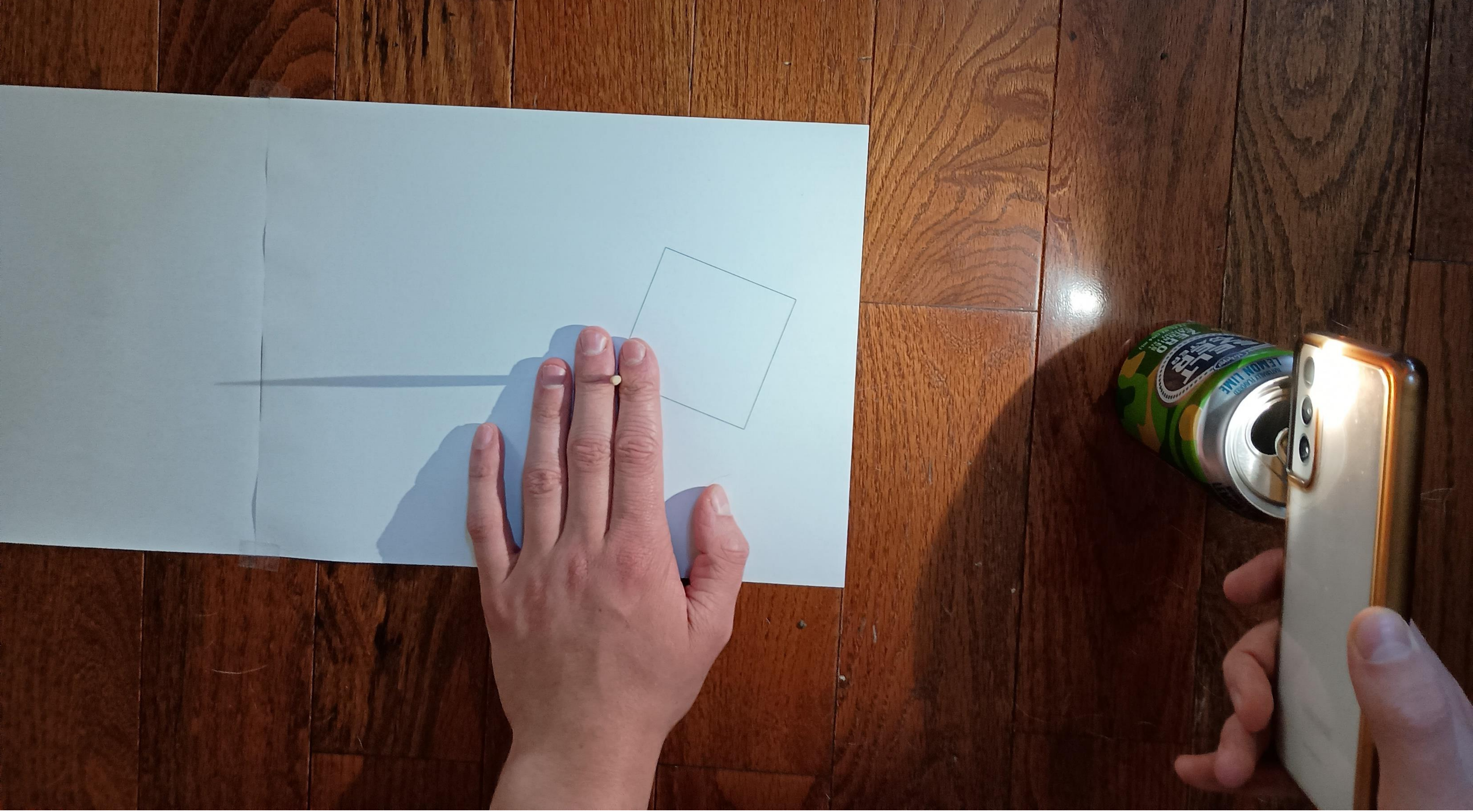


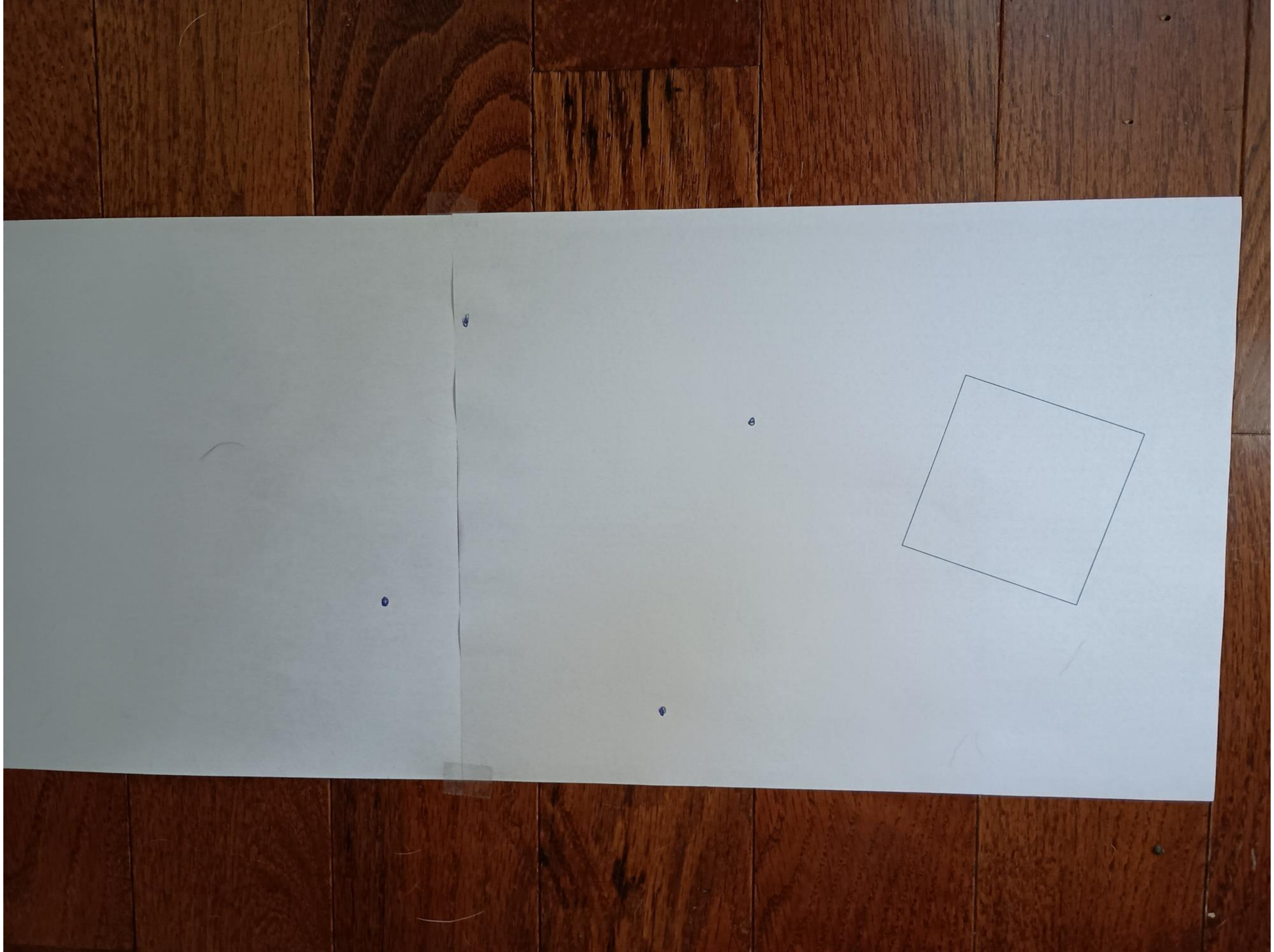
Handout is available at
https://drive.google.com/drive/folders/1mtgz72ZColcLCEznaxEnuLPa1-7Yfraw?usp=sharing

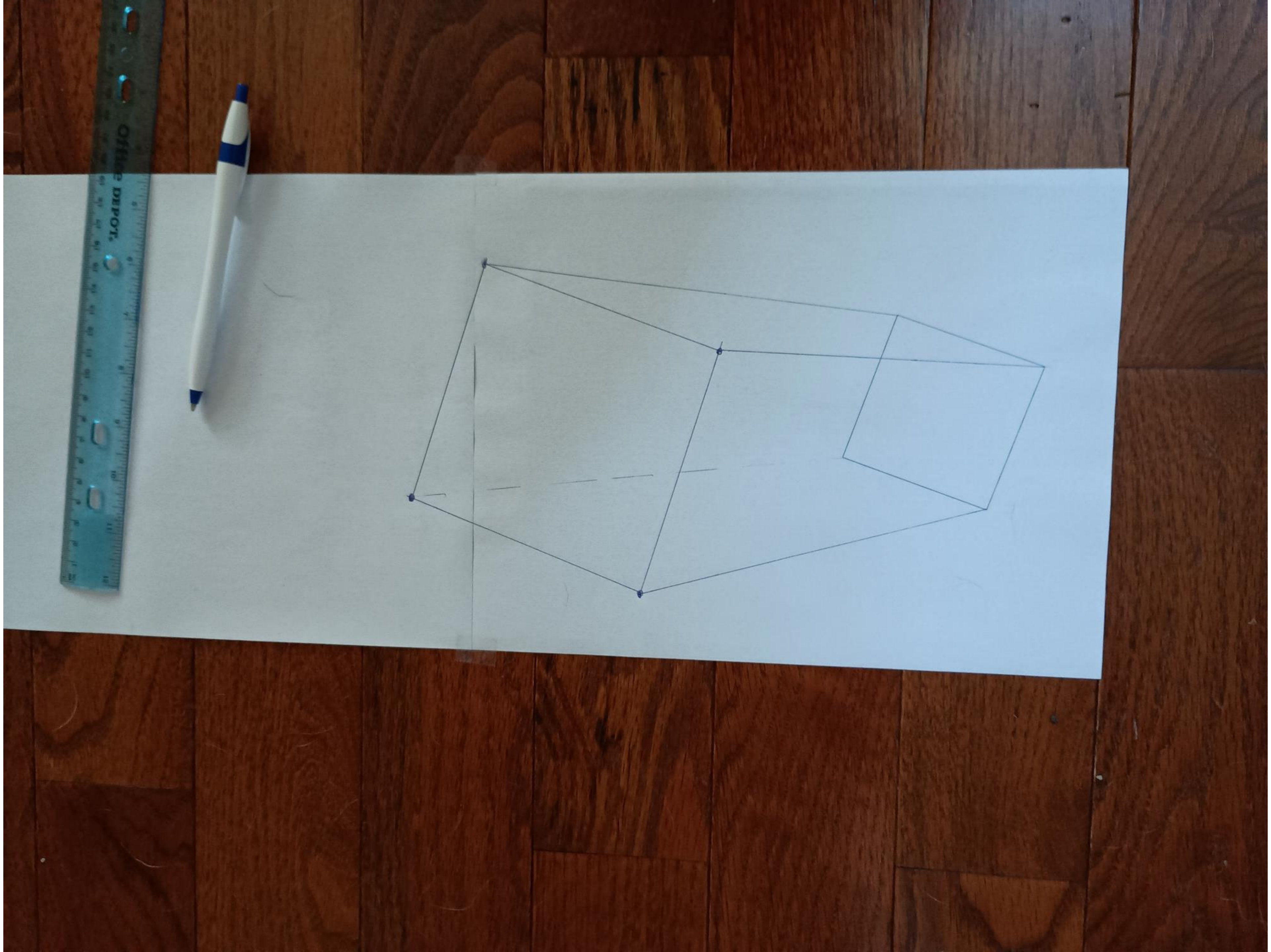
Shady Business:
making anamorphic art with shadows

Materials




LEMON LIME




Office DEPOT.

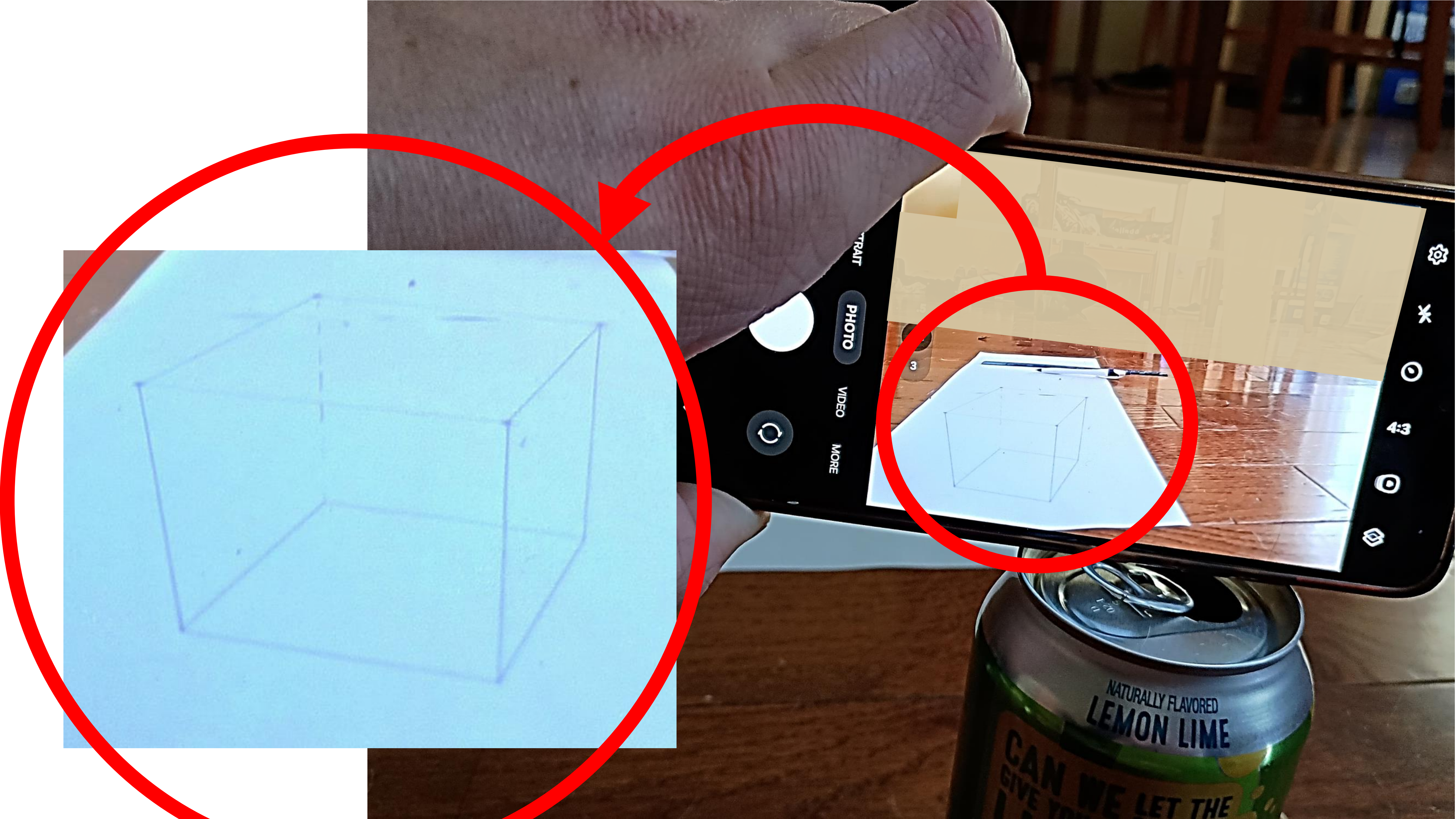
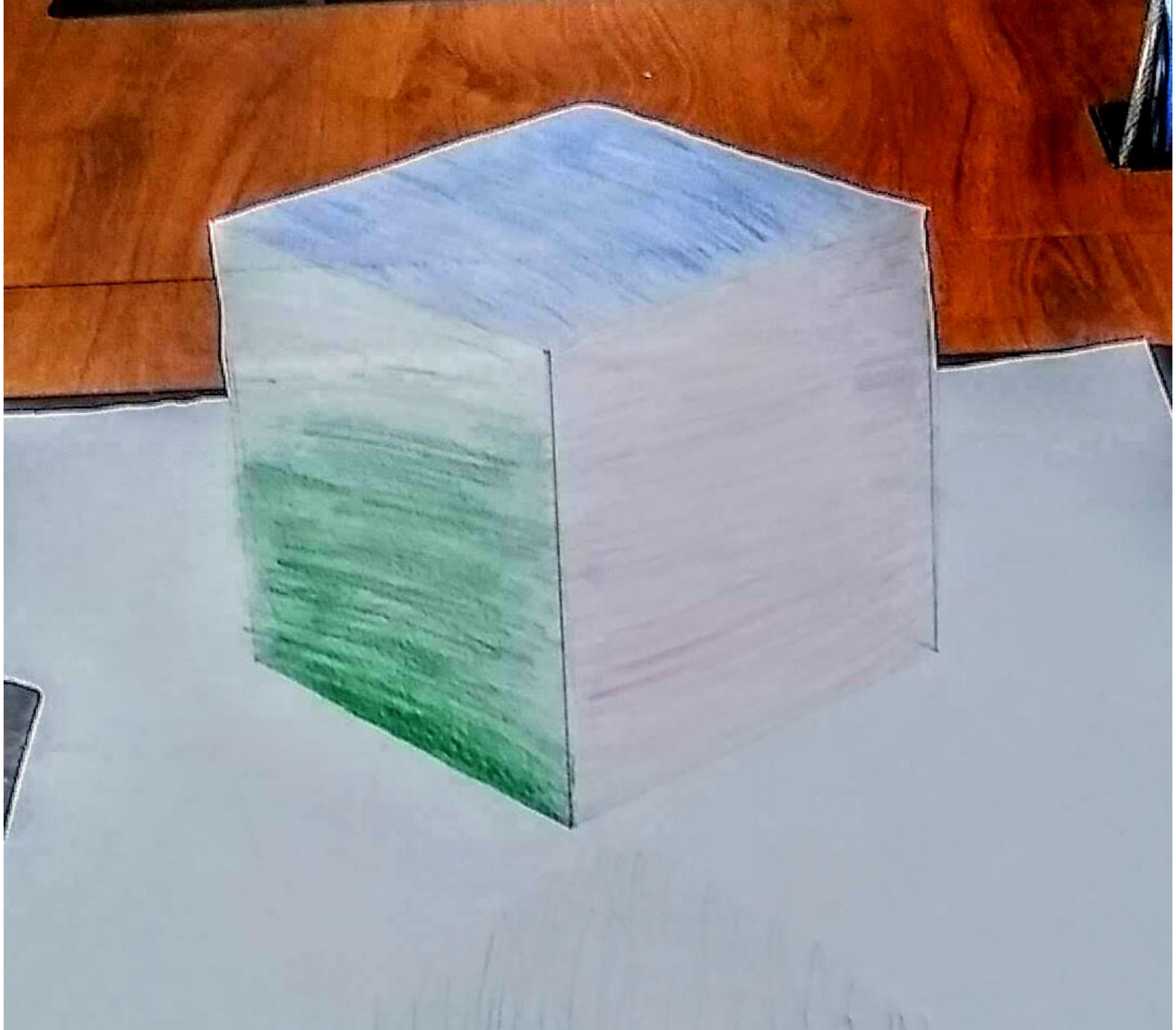


PHOTO
VIDEO
MORE
4:3
NATURALLY FLAVORED
LEMON LIME

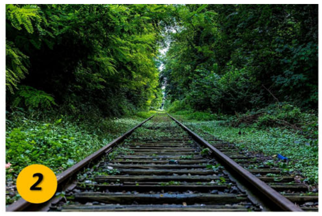


Where was Eye? (part 1)

Exploration 1. Three students, Adam, Benjamin, and Cayla, took part in a photography competition. All three submitted photos of railroad tracks. Adam and Benjamin took photos from their natural height; Cayla flew a drone high above her head to take a picture.

(A) Adam (B) Benjamin (C) Cayla

The images Adam, Benjamin, and Cayla took appear below (in no particular order).



Match each image with the photographer who took it.

Photo 1 was taken by

Adam

Benjamin

Cayla

? Check work

Photo 2 was taken by

Adam

Benjamin

Cayla

? Check work

Photo 3 was taken by

Adam

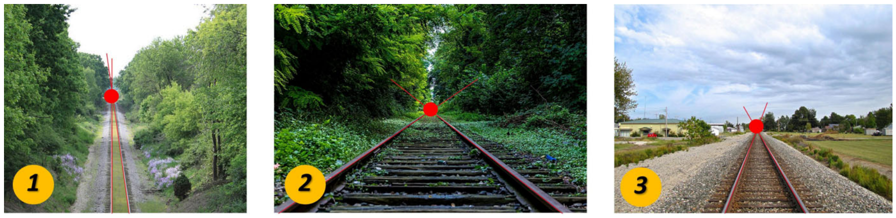
Benjamin

Cayla

? Check work

Group Discussion Prompt: *Discuss the reasons for your choices. Do you think it is possible to use these photographs to estimate the height of the camera that took each photo?*

Exploration 2. Let's look at the three photos geometrically.

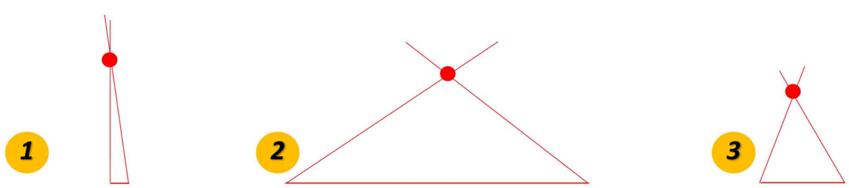


The rails appear to meet at a single point. Do you know what this point is called?

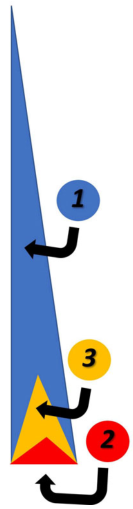
- Center point
- Vanishing point
- Vertex
- End point

? Check work

The rails, together with the bottom of each photo, form triangles.



You might describe the first triangle as tall and narrow, and the second triangle as short and wide. These descriptions refer to the *proportions* of these triangles. To really see the difference in the proportions, we can make the bases of the triangles the same size. This makes sense to do because railroad tracks have the same width everywhere in the U.S.



Group Discussion Prompt: *In the previous exploration, you figured out who took what photo. Discuss the relationship between the height of the camera and the height of the triangles. Click on the arrow (below, right) for a hint.*

To see the change in the height of the triangle as the camera height changes, you can use your own phone camera. Find a long hallway or a long table. Position your camera in the middle of the hallway (table). Move the camera up and down observing how the edges of the hallway (table) create taller and shorter triangles, as shown in the photos below.

As the camera lowers, the triangle becomes proportionally shorter.
As the camera lowers, the triangle becomes proportionally taller.
As the camera lowers, the triangle's proportions do not change.

? Check work

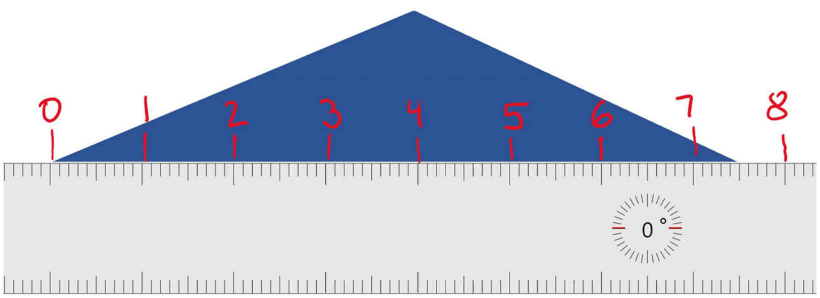
So far, we have established that there is a relationship between the proportions of the triangle formed by rails (or edges of a hallway), and the height of the camera that took the photo. Next, we will look at a photo for which the measurements of the real-life hallway, and the height of the camera are known. We will numerically relate the proportions of the triangle to the height of the camera.

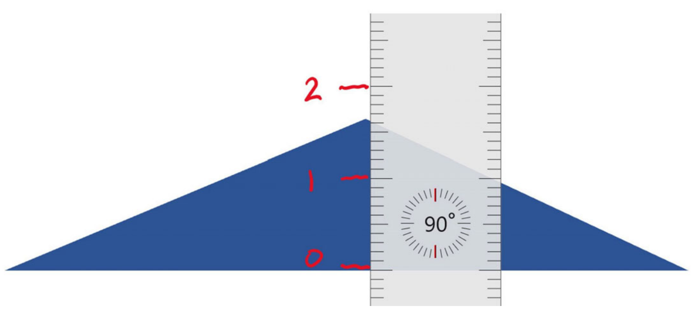
Exploration 3.

The following photo shows a 92-inch wide hallway. The photo was taken with the camera lens 20 inches above the floor.

Observe that the resulting triangle is nearly isosceles. We achieved this by placing the camera as close to the middle of the hallway as we could.

We will now look at the ratio of the height of the triangle to the length of the base.

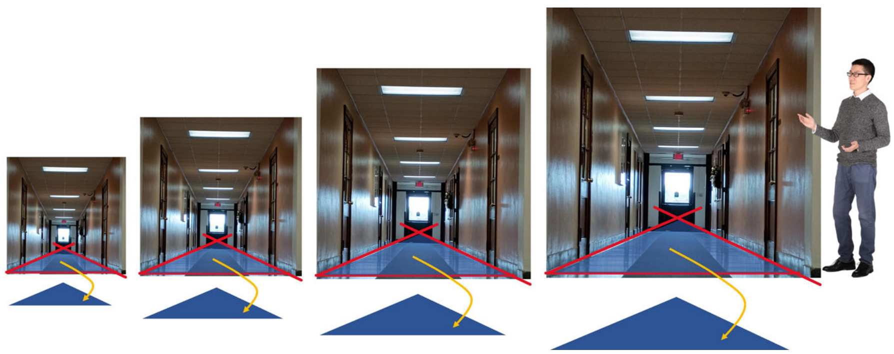




$$\frac{\text{height}}{\text{base}} = \frac{1.65}{7.5} = 0.22$$

Group Discussion Prompt: *Note that our calculations omitted units. Do units matter in this case? If we were to do our measurements using different units, would the ratio still be the same?*

Next, we will try to figure out what to do with this ratio to find an estimate for the height of the camera. To do this, suppose we made four prints of our photo. The first print is the size of a small poster, the last print is life-sized. The resulting triangles are clearly of different sizes, but they have the same *proportions*! Such triangles are called — ?



Since the last "print" in the above graphic is life-sized, and the width of the real-life hallway is 92 inches, we can conclude that the base of the last triangle is 92 inches. Set up and solve a proportion to find the height of the last blue triangle.

$$\frac{1.65}{7.5} = 0.22 = \frac{\text{height of last triangle (inches)}}{\boxed{}\ ?\ \text{in}}$$

Height of last triangle = ? in

Compare your answer to the height of the camera given at the start of the problem. The numbers are very close! Do you think this is a coincidence?

Group Discussion Prompt: *Formulate a rule for finding the height of the camera for photos with the same set up. How would you verify that your rule works for ALL such photos? Discuss this with your classmates and your teacher.*

Group Discussion Prompt: *The rule you stated in the previous discussion question is a theoretical rule. In practice, the height of the camera above the floor was given to be* 20 *inches, but the computed height of the triangle was not exactly* 20. *What do you think accounts for the* *difference?*

Group Discussion Prompt: *Railroads in the U.S. are built so that the distance between the rails is the same everywhere in the country. The distance between the rails is call the gauge. You can learn more about railroad gauges* here. *Discuss how you can use this information to find the actual heights that Adam, Benjamin, and Cayla took their pictures from.*

In the next two parts of this activity, you will get to perform a photo experiment yourself, and develop the theoretical underpinnings for the rule you discovered here.

Photo Credits

Railroad track photos used in this activity were downloaded from WikiMedia Commons.

Photo 1: Dwight Burdette, CC BY 3.0 ¡https://creativecommons.org/licenses/by/3.0¿, via Wikimedia Commons

Photo 2: Antoine Beauvillain, CC0, via Wikimedia Commons

Photo 3: Brian Stansberry, CC BY 4.0 ¡https://creativecommons.org/licenses/by/4.0¿, via Wikimedia Commons

Microsoft clip art was used for student photos.

← Previous Next → 2024-08-05 16:44:14

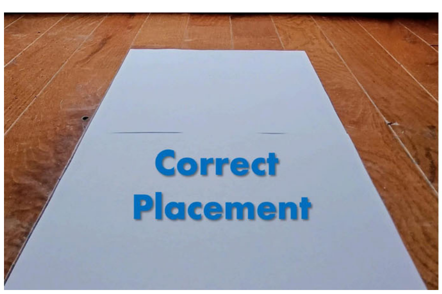
Where was Eye? (part 2)

In this part of the activity, you will take your own photos from a known height to test the formula you had developed in Part 1.

Directions

In groups of two or three, you will use a phone to take photos of a long desk, a sheet of paper or a hallway while holding your phone at a known height.

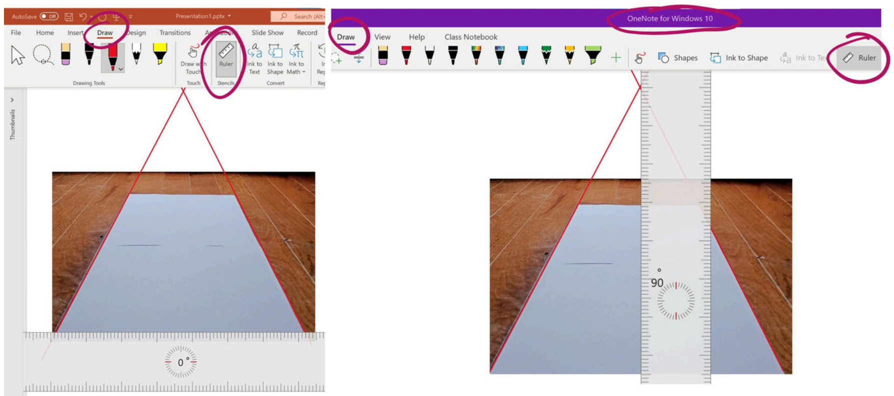
- Make sure that the camera is located in the center of the hallway/desk/paper so that the triangle formed by the edges is isosceles.



- To keep track of the camera height, hold or tape the phone to a meter stick or a ruler. Have one group member record the vertical distance from the surface (hallway floor/desktop/paper) to the camera lens for every shot. Remember to measure the distance to the camera lens (not the bottom or the top of the phone).

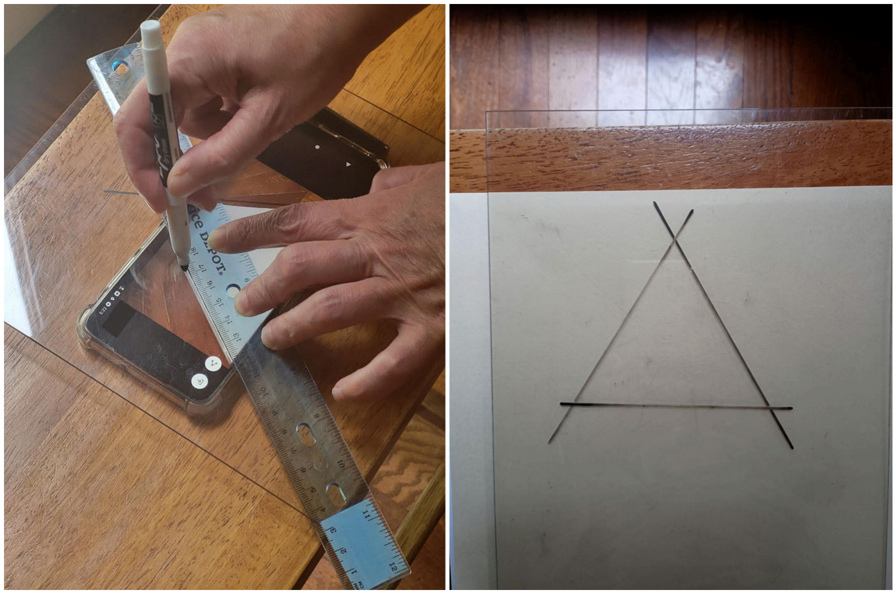
- Below are two methods for drawing and measuring the triangle.

  (a) Method 1. If you have a touch-screen computer, import your photos into PowerPoint (left) or OneNote (right). Use the ruler tool (under "Draw") to outline the edges of the hallway/desk/paper in each photo. Form a triangle with the vanishing point as the top vertex, as shown below.



Measure the length of the base, and the height of the triangle and record your measurements.

(b) Method 2. You can draw the triangle directly on your phone photo by overlaying a piece of Plexiglass over your photo and using a dry-erase marker and a ruler to trace and extend the edges of the hallway/desk/paper to form a triangle, as shown below (left).



You can now do your measurements on the tracing (right).

- Follow the procedure you developed in Part 1 to find the height of the camera using ratios. Compare your computed height to your measured height. How close did you get?

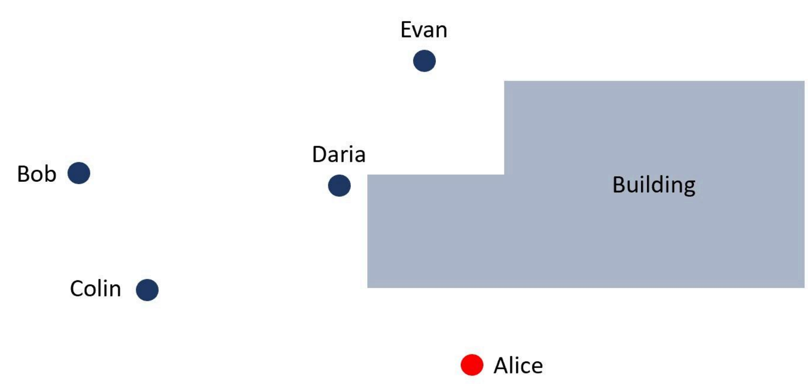
Remark. If done carefully, these photo experiments typically produce good results. If your relative error, $\left(\frac{|\text{measured height} - \text{computed height}|}{\text{measured height}}\right)$, is greater than five percent, consider the following sources of error:

- Did you set up your ratios correctly? Did you solve the equation correctly?
- Is your triangle nearly isosceles?
- Did you measure the vertical distance to the actual camera lens? (It is a common mistake

Where was Eye? (part 3)

We will now develop a theoretical foundation for our method of figuring out the height of the camera.

Exploration 1. Suppose Alice, Bob, Colin, Daria, and Evan are playing hide-and-seek in the school yard. Alice is the seeker. The diagram below shows the location of the players. Which of the children is visible to Alice?



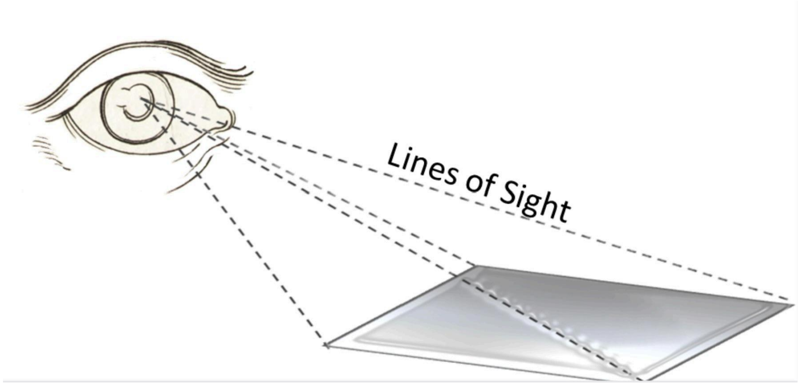
Check the names of all the children that Alice can see.

- Bob
- Colin
- Daria
- Evan

? Check work

Group Discussion Prompt: *Articulate the reason for your choices. If you need help formulating your thoughts, click on the arrow (below, right), and use the diagram to help you.*

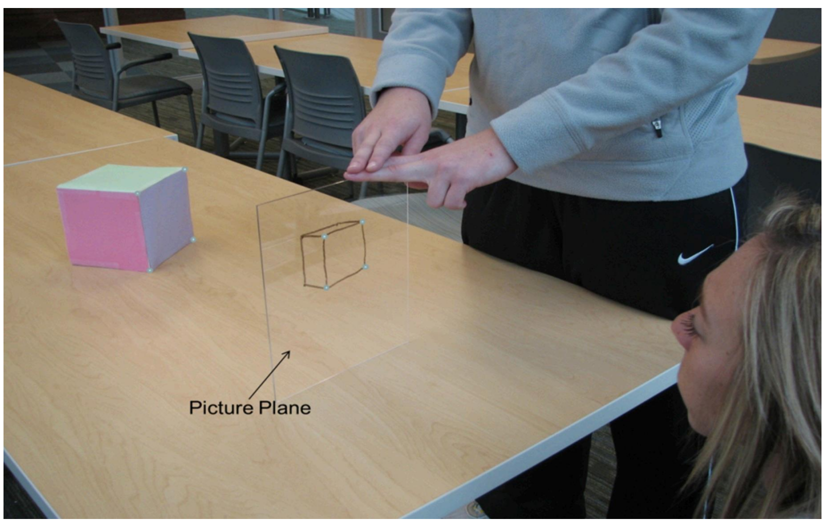
The above exploration intuitively established a very important fact: we see along straight lines. These lines are called *lines of sight*.



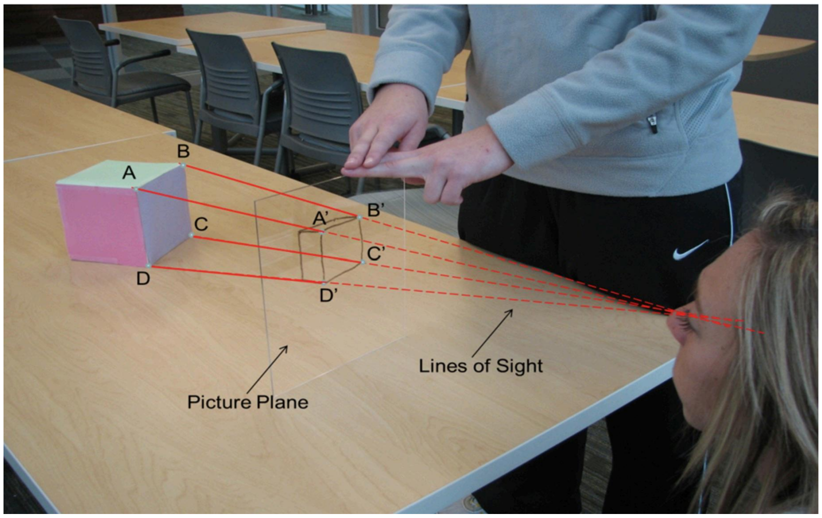
Picture Planes

To understand how lines of sight can help us create a realistic picture, imagine a canvas made of glass. If you position the glass canvas in front of the object you want to draw, you can simply trace the object onto the glass with a marker. We will call the glass canvas a *picture plane*.

Take a look at the photograph below. The student in the photo has just finished tracing the cube onto the glass. From her point of view, the tracing matches up with the cube.



Now let’s draw lines of sight that connect the corners of the cube with the eye. The line of sight from each corner of the cube passes through its image on the glass!

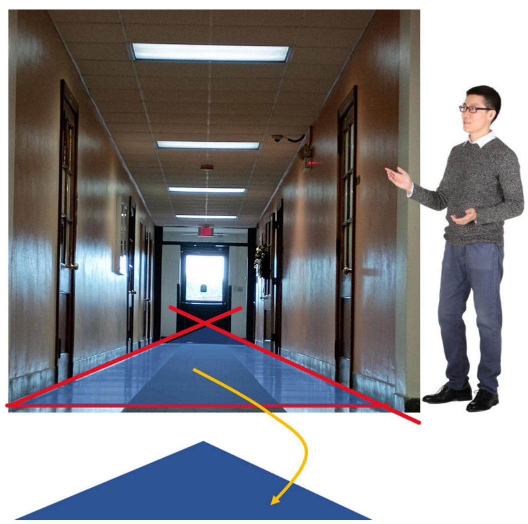


This principle allows artists and computer programmers to draw any object from the point of view of an imaginary eye. If we were to place a camera where the student's eye was located, the picture taken by the camera would match the tracing on the glass.

Vanishing Point and the Height of the Eye (Camera)

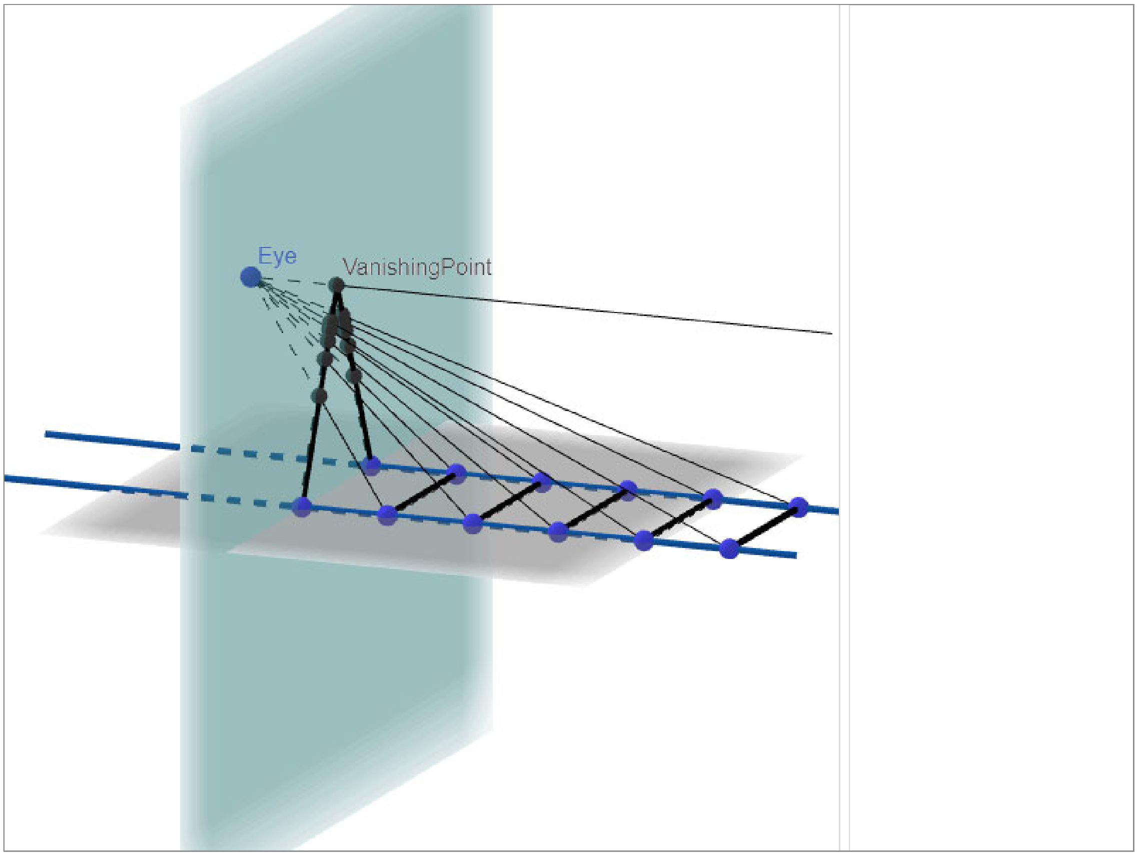
Now that we know about lines of sight and the picture plane, we are ready to figure out why the method we discovered and tested in the first two parts of the activity actually works.

Exploration 2. Recall that when a life-sized version of an image was used, the height of the triangle was equal to the height of the camera.



The interactive model below shows railroad tracks and the Eye (camera) looking at the tracks. The vertical plane is a picture plane. The triangle in the picture plane is the image of the rails the Eye sees. Note the location of the vanishing point. Observe how the sides of the triangle are formed by points of intersection of lines of sight with the picture plane. You can rotate the model for a better view. Use the slider to adjust the height of the Eye and note how the image in the picture plane changes.

RIGHT-CLICK and DRAG to rotate the model for a better view.



Group Discussion Prompt: *Explain why the height of the eye is the height of the triangle.*

2024-08-05 16:44:26

Lines of Sight

an interdisciplinary exhibition by Anna Davis

2016

Contents

Chapter 1: The making of an exhibition

This scrapbook contains all posters and other artifacts associated with the *Lines of Sight* exhibition at *The Works* in Newark, OH. The exhibition ran from January 29th to April 10th of 2016. *The Works* is a Smithsonian Affiliate Institution. According to the website, "The Works: Ohio Center for History, Art & Technology is an interactive museum for families of all ages."

The exhibition was designed to provide a hands-on, interactive experience for children and adults who want to explore mathematics of visual perspective, art, computer graphics, history, and our perception of the visual world. The exhibition was sponsored by *Park National Bank*; cardboard was contributed by *PCA Packaging Corporation of America.*

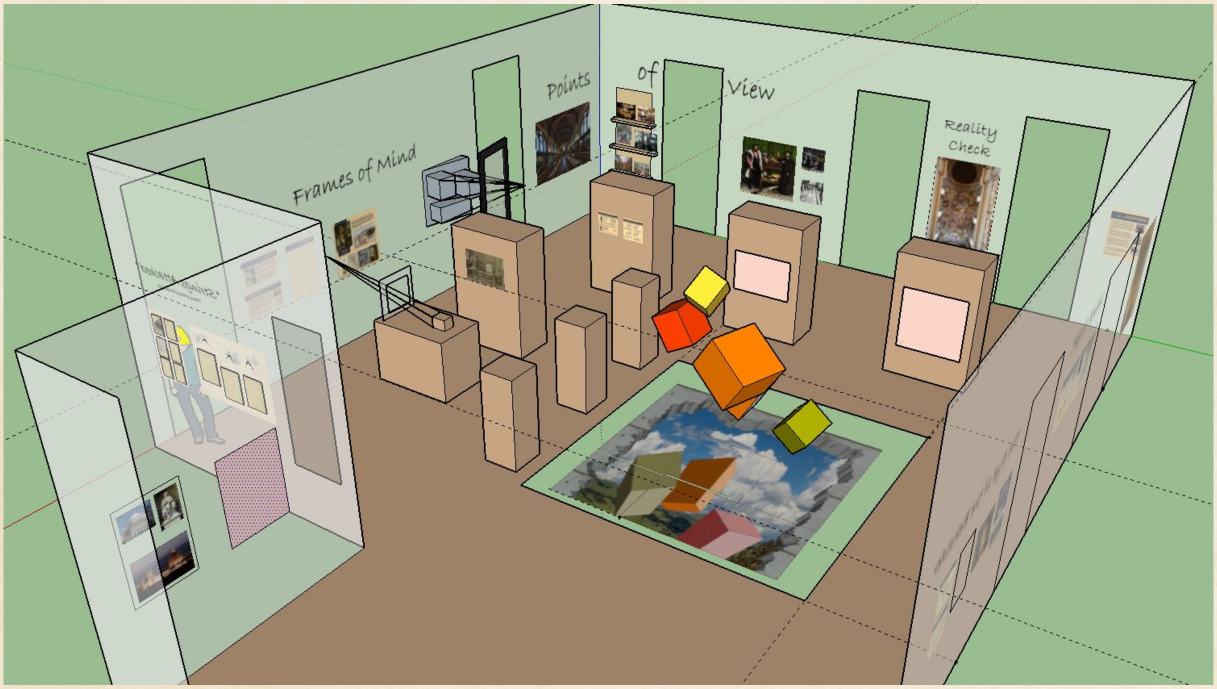


Figure 1.1: *Lines of Sight* concept proposal. SketchUp 3D model by Anna Davis.

Figure 1.2: Final installation.

Exhibition Stations

The exhibition was broken up into multiple stations, each addressing a narrow area of interest. By traversing the gallery in the clockwise direction, visitors got acquainted with the history, mathematics, and applications of visual perspective. The following table lists the exhibition stations and their descriptions.

Station title	Description
Strings Attached	Discusses the nature of light and early investigations of visual perspective by Leonardo and Desargues.
Frames of Mind	Introduces the concept of a persepctive frame. Contains a working replica of Dürer's perspective frame. Discusses computer graphics and how visuals can affect the brain.
Points of View	Contains interactive exercises and a discussion of the importance of viewing location.
Reality Check	Contains examples of early anamorphic art, virtual architecture, and anamorphic street art.
Eye of the Beholder	Contains a working camera obscura and a discussion on the mechanics of the eye.
Hind Sight	A discussion of how photography and mathematics of straight lines can be used to figure out what happened in the past.

All posters were designed by Anna Davis. Images used in the posters are either in the public domain or were used by permission. Photos of the exhibition are courtesy of *The Works*, Anna Davis, and Tom Brockman.

Station 1: Strings Attached

The aesthetic of this three-dimensional display was inspired by illustrations of Desargues' manuscript on geometry of visual perspective. In Desargues' tretease, lines of sight were depicted as strings, as seen in the posters below.

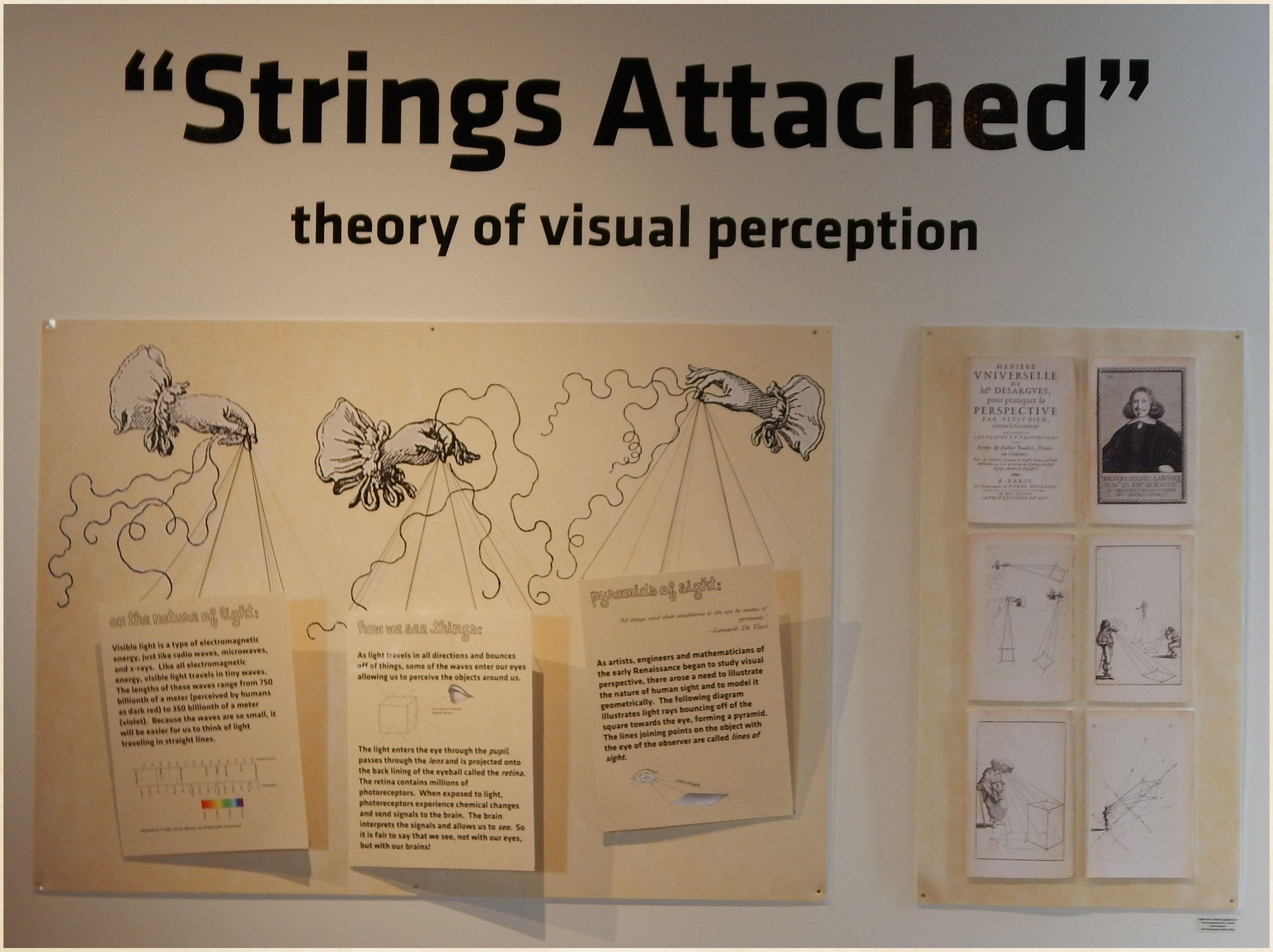


Figure 1.3: *Strings Attached* display lays the geometric foundation of the theory of perspective.

Station 2: Frames of Mind

This section introduces the concept of a picture plane by considering the use of various types of perspective frames throughout history. A functional replica of Dürer's perspective frame was created by volunteers Col. Bill Snider, Deb Tung and Harvey Tung.

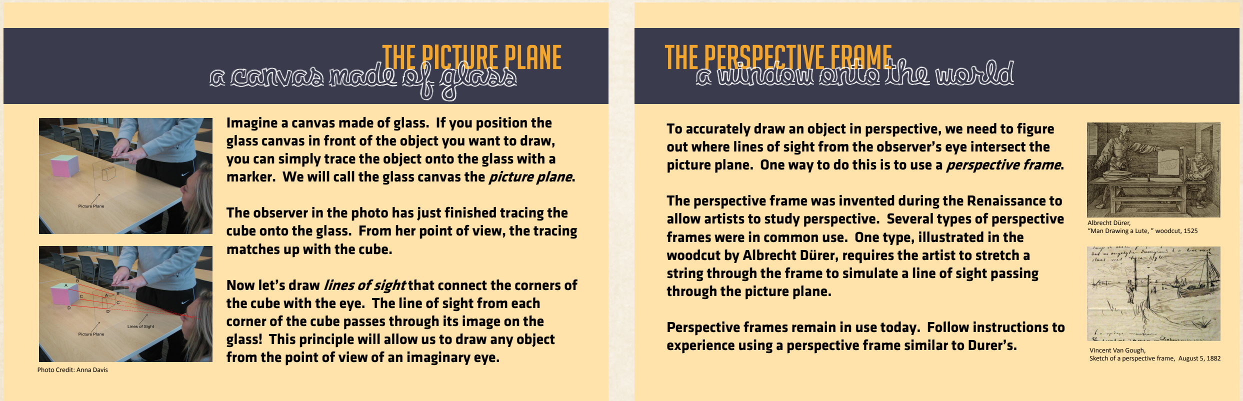


Figure 1.4: The poster on the left features ODU math students using plexiglass as a picture plane to illustrate that lines of sight, which connect points on the object with the corresponding points on the tracing, converge at the eye. The poster on the right shows two perspective frames: one used by Dürer, and one used by Van Gough.

Figure 1.5: A working replica of Dürer's perspective frame was built by Col. Bill Snider, Deb Tung, Harvey Tung, and Anna Davis. The poster on the left provides step-by-step directions for how to use the frame to draw the wooden house. A copy of Dürer's woodcut which inspired this contraption can be seen behind the frame in the middle photo and in Figure 1.4.

Figure 1.6: Albrecht Dürer and his art.

Next, we use the concept of the picture plane to develop the mathematics behind computer graphics.

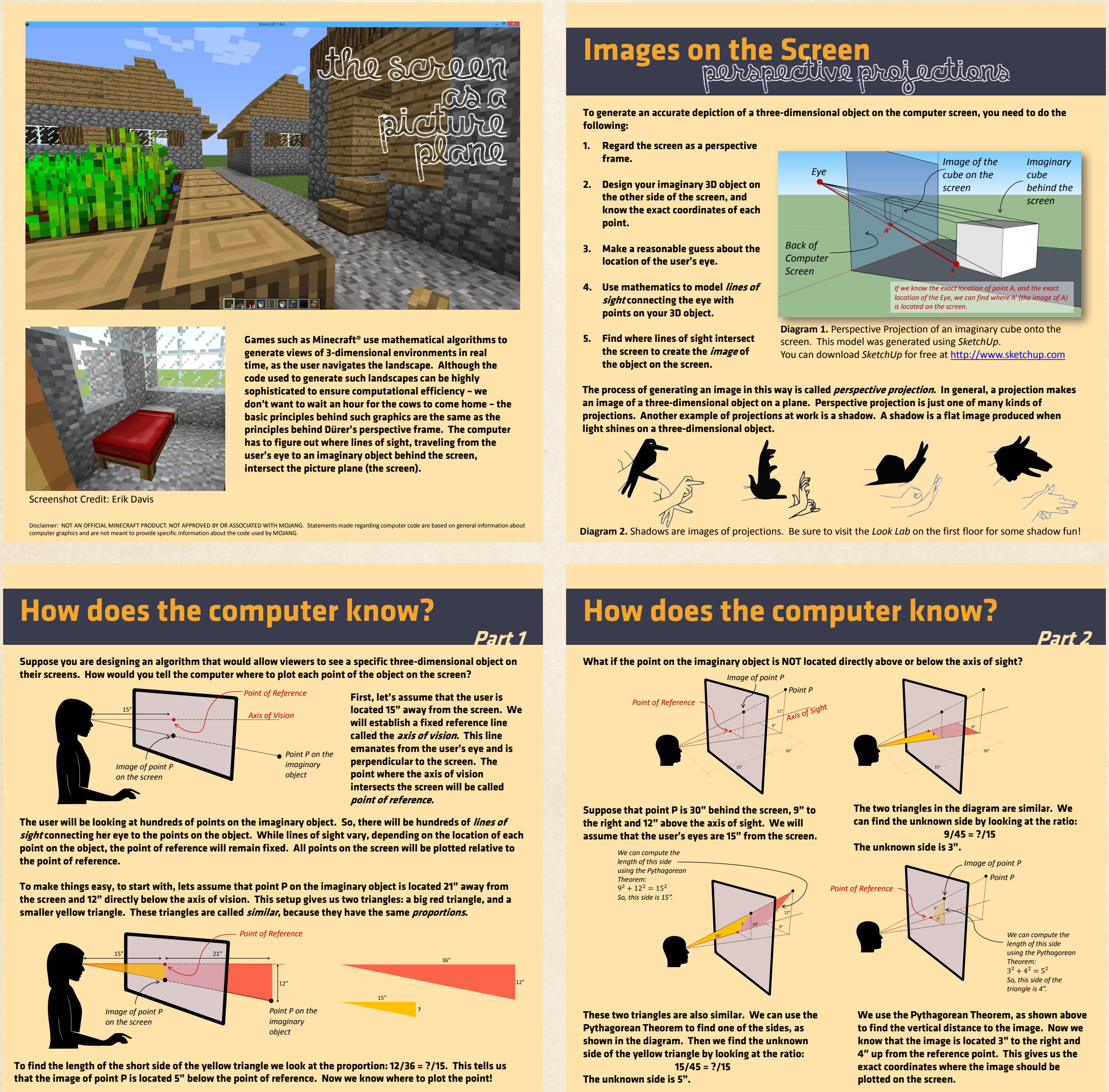


Figure 1.7: Using familiar virtual environments such a Mincraft, this set of posters sets the scene for the calculations involved in generating computer graphics.

Station 3: Points of View

In this section we start to explore how the viewer's vantage point affects the viewer's perception of the image.

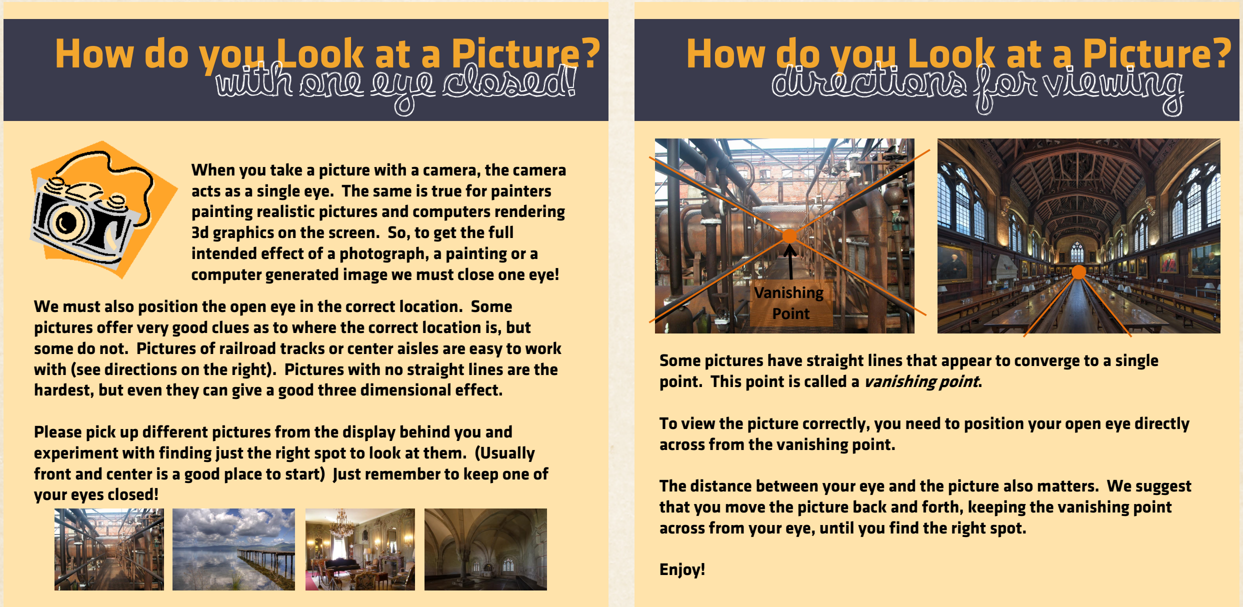


Figure 1.8: These posters discuss how one should look at an image to experience the full perspective effect.

Figure 1.9: A large poster of Balliol College, and multiple smaller prints allow visitors to practice locating the vanishing point and using it to find the perfect vantage point from which to experience each print.

We wrap up this section by looking at how simulated environments can affect the brain.

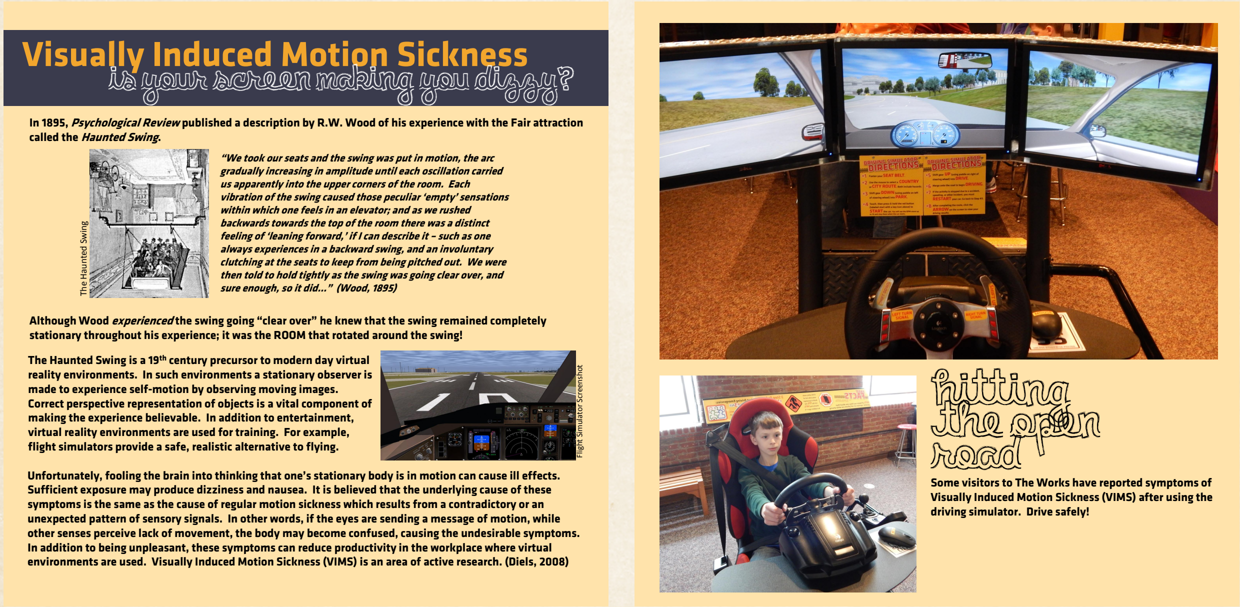


Figure 1.10: These posters explore historical and modern environment simulators and their potential effect on the brain.

Station 4: Reality Check

At this station viewers explore how artists can intentionally create images that require a certain (unusual) vantage point to be viewed correctly. The resulting pieces are classified as *anamorphic art*. Throughout history, anamorphic art was used to create hidden images, generate fun effects, and to enhance architectural features.

Figure 1.11: Hans Holbein's famous piece *The Ambassadors* contains a distorted image of a skull in the foreground. When viewed from an extreme angle from the side - as the boy is doing in the photo - the skull appears to pop out of the canvas.

Figure 1.12: Hans Holbein the younger and his art.

Using anamorphic art to create "fake" architectural features, such as the domes below, became a sought-after signature skill of Andrea Pozzo.

Figure 1.13: These posters show how anamorphic art can be used to create fake spaces or to enhance architectural features.

Anamorphic street art is a popular modern art form. Some artists use their intuition alone, while others use mathematical principles to create their work.

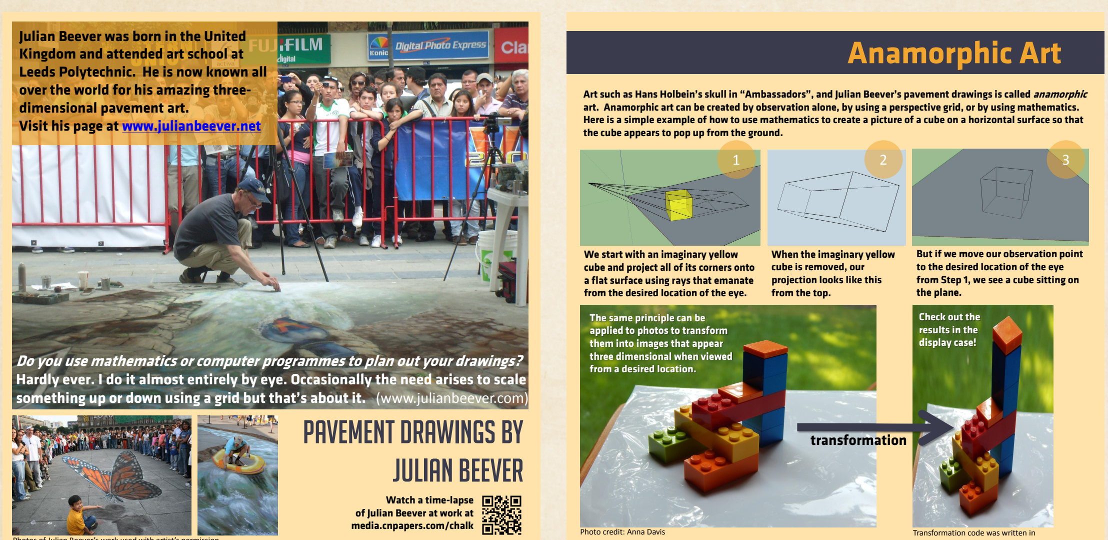
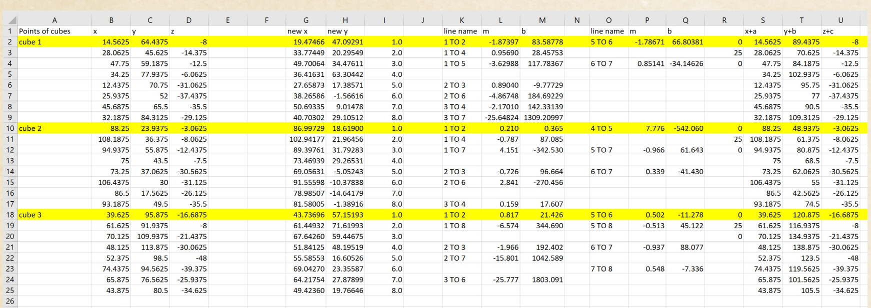


Figure 1.14: The poster on the left features the art of Julian Beever. The poster on the right features a photograph by Anna Davis distorted using an algorithm to create the display in Figure 1.15. The distortion algorithm was written by Anna Davis using Wolfram Mathematica.

Figure 1.15: A flat, distorted lego photo from Figure 1.14 is displayed on the left. The top portion of the photo had been cut away. When viewed from the correct vantage point, the legos in the flat photo appear three-dimensional (center photo). Young visitors were captured enjoying the anamorphic effect (right).



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Points of cubes	x	y	z			new x	new y			line name	m	b		line name	m	b		x+a	y+b	z+c
2	cube 1	14.5625	64.4375	-8			19.47466	47.09291	1.0		1 TO 2	-1.87397	83.58778		5 TO 6	-1.78671	66.80381	0	14.5625	89.4375	-8
3		28.0625	45.625	-14.375			33.77449	20.29549	2.0		1 TO 4	0.95690	28.45753					25	28.0625	70.625	-14.375
4		47.75	59.1875	-12.5			49.70064	34.47611	3.0		1 TO 5	-3.62988	117.78367		6 TO 7	0.85141	-34.14626	0	47.75	84.1875	-12.5
5		34.25	77.9375	-6.0625			36.41631	63.30442	4.0										34.25	102.9375	-6.0625
6		12.4375	70.75	-31.0625			27.65873	17.38571	5.0		2 TO 3	0.89040	-9.77729						12.4375	95.75	-31.0625
7		25.9375	52	-37.4375			38.26586	-1.56616	6.0		2 TO 6	-4.86748	184.69229						25.9375	77	-37.4375
8		45.6875	65.5	-35.5			50.69335	9.01478	7.0		3 TO 4	-2.17010	142.33139						45.6875	90.5	-35.5
9		32.1875	84.3125	-29.125			40.70302	29.10512	8.0		3 TO 7	-25.64824	1309.20997						32.1875	109.3125	-29.125
10	cube 2	88.25	23.9375	-3.0625			86.99729	18.61900	1.0		1 TO 2	0.210	0.365		4 TO 5	7.776	-542.060	0	88.25	48.9375	-3.0625
11		108.1875	36.375	-8.0625			102.94177	21.96456	2.0		1 TO 4	-0.787	87.085					25	108.1875	61.375	-8.0625
12		94.9375	55.875	-12.4375			89.39761	31.79283	3.0		1 TO 7	4.151	-342.530		5 TO 7	-0.966	61.643	0	94.9375	80.875	-12.4375
13		75	43.5	-7.5			73.46939	29.26531	4.0										75	68.5	-7.5
14		73.25	37.0625	-30.5625			69.05631	-5.05243	5.0		2 TO 3	-0.726	96.664		6 TO 7	0.339	-41.430		73.25	62.0625	-30.5625
15		106.4375	30	-31.125			91.55598	-10.37838	6.0		2 TO 6	2.841	-270.456						106.4375	55	-31.125
16		86.5	17.5625	-26.125			78.98507	-14.64179	7.0										86.5	42.5625	-26.125
17		93.1875	49.5	-35.5			81.58005	-1.38916	8.0		3 TO 4	0.159	17.607						93.1875	74.5	-35.5
18	cube 3	39.625	95.875	-16.6875			43.73696	57.15193	1.0		1 TO 2	0.817	21.426		5 TO 6	0.502	-11.278	0	39.625	120.875	-16.6875
19		61.625	91.9375	-8			61.44932	71.61993	2.0		1 TO 8	-6.574	344.690		5 TO 8	-0.513	45.122	25	61.625	116.9375	-8
20		70.125	109.9375	-21.4375			67.64260	59.44675	3.0									0	70.125	134.9375	-21.4375
21		48.125	113.875	-30.0625			51.84125	48.19519	4.0		2 TO 3	-1.966	192.402		6 TO 7	-0.937	88.077		48.125	138.875	-30.0625
22		52.375	98.5	-48			55.58553	16.60526	5.0		2 TO 7	-15.801	1042.589						52.375	123.5	-48
23		74.4375	94.5625	-39.375			69.04270	23.35587	6.0						7 TO 8	0.548	-7.336		74.4375	119.5625	-39.375
24		65.875	76.5625	-25.9375			64.21754	27.87899	7.0		3 TO 6	-25.777	1803.091						65.875	101.5625	-25.9375
25		43.875	80.5	-34.625			49.42360	19.76646	8.0										43.875	105.5	-34.625
26																					
27	EYE	60	-96	66																	

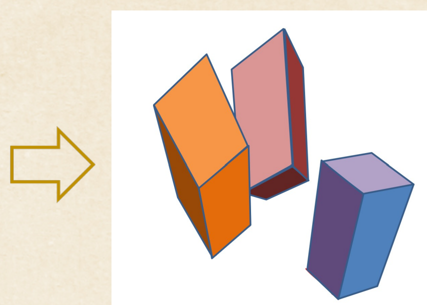


Figure 1.16: A large anamorphic centerpiece was installed in the middle of the gallery. This proved to be a logistical nightmare, as the cubes kept falling and setting off alarms in the middle of the night. The spreadsheet contains the actual calculations used to create the distorted image of the cubes.

Station 5: Eye of the Beholder

> The camera obscura mimics the workings of the human eye and a photographic camera. This station explores the physics and psychology of sight.

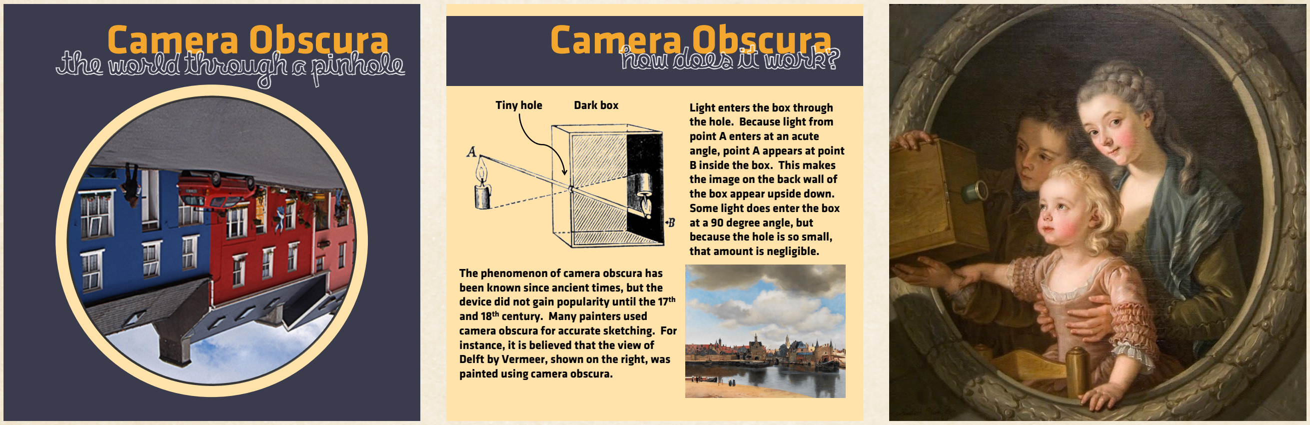


Figure 1.17: The physics and uses of camera obscura.

Figure 1.18: A working camera obscura, built by Col. Bill Snider and Anna Davis, allowed visitors to view cars drive by the gallery window (upside-down and backwards!) by putting their heads inside the box with a tiny pin-hole.

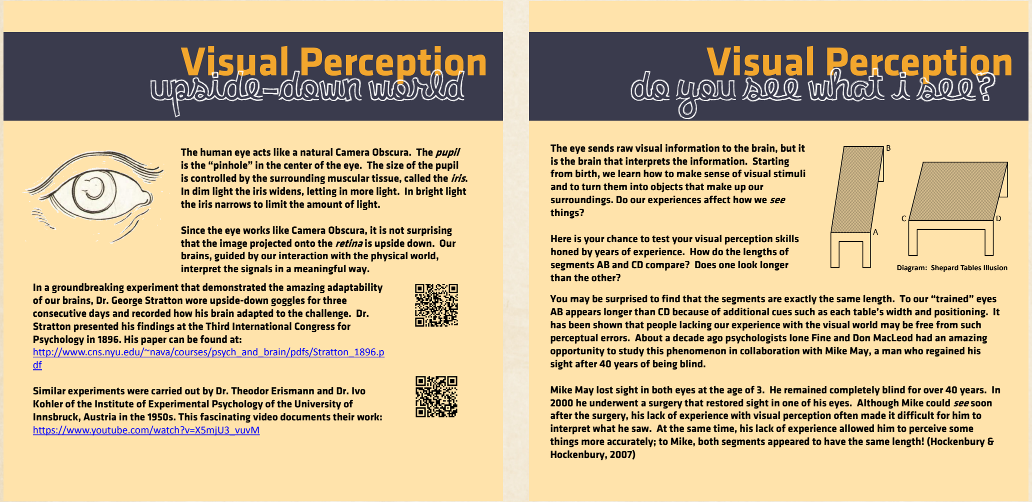


Figure 1.19: These posters discuss some interesting elements of the psychology of sight.

What the audience learns from working with lines of sight can be easily applied to other linear phenomena. The following example describes an investigation performed by the *Time Scanners* team to solve a World War II mystery.

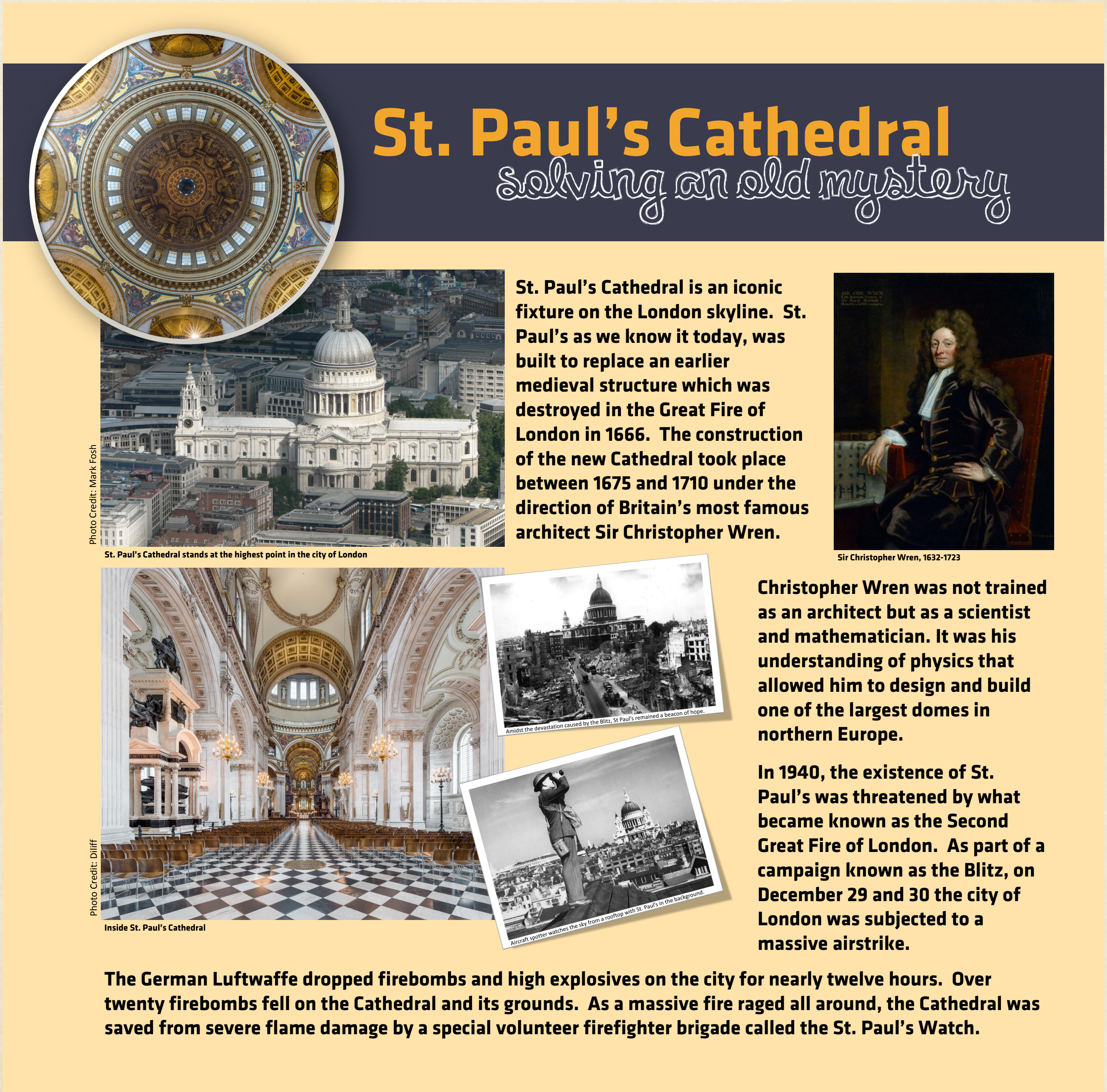


Figure 1.20: St. Paul's Cathedral story.

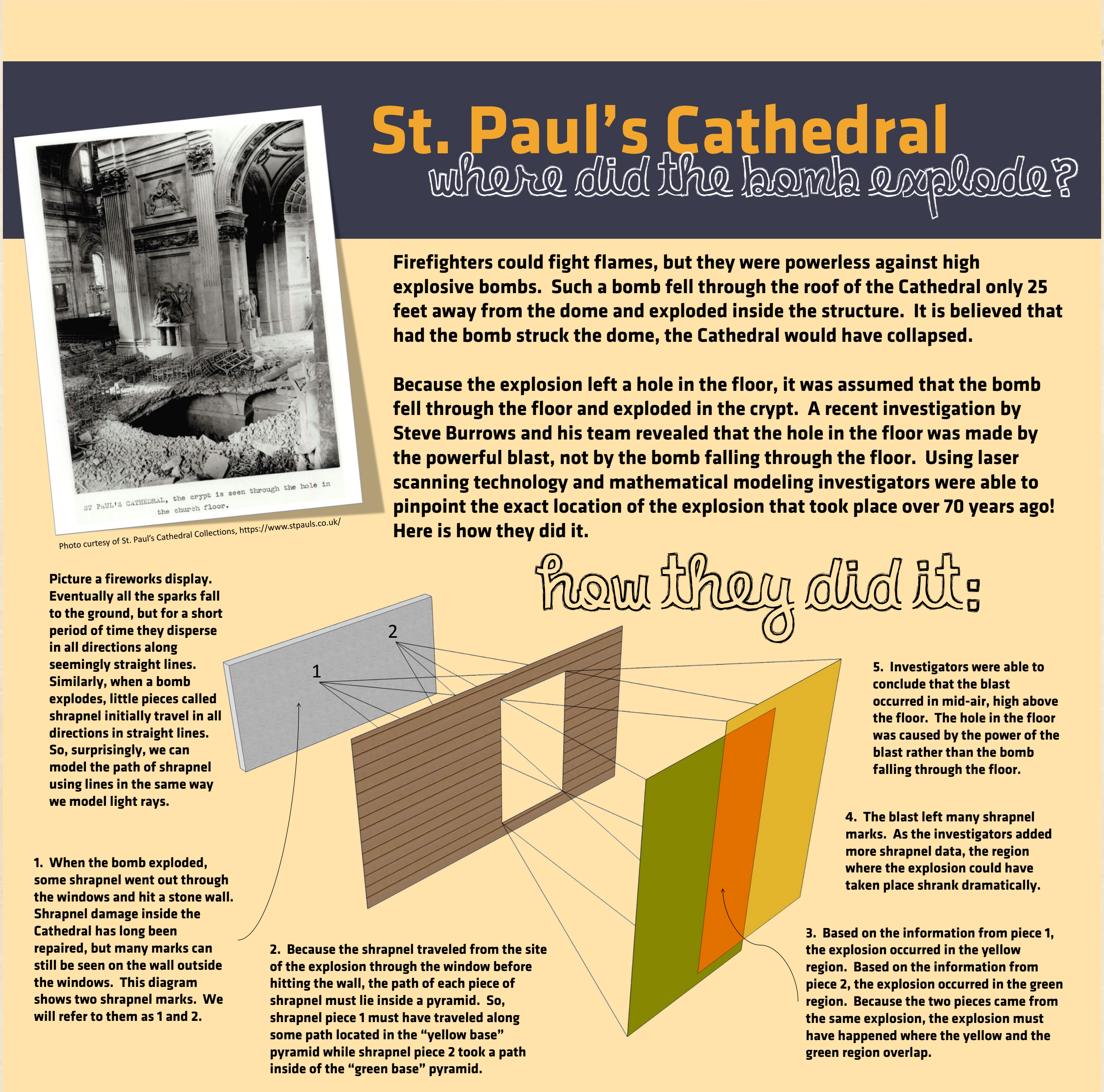


Figure 1.21: Solution to the St. Paul's Cathedral mystery.

Just like knowing the location of the eye helps us construct accurate perspective renderings, working backwards from an accurate image can help us deduce the location of the eye. Working backwards from a photograph to figure out sizes and distances is sometimes called forensic photography. The following set of posters illustrates this topic.

Figure 1.22: A fairy tale that illustrates forensic photography.

No historical study of perspective would be complete without a mention of Filippo Brunelleschi. The following poster was located at the entry/exit to the exhibition.

Figure 1.23: Filippo Brunelleschi and his perspective experiment.

Acknowledgements

A very special thank you goes to Col. Bill Snider, Deb Tung and Harvey Tung for making very sturdy, aesthetically pleasing and functional props: the perspective frame, the wooden house, and the camera obscura.
I would also like to thank my son, Erik for suggestions, and Minecraft screenshots.
I am also grateful to the following organizations:

- Park National Bank for sponsoring the exhibition
- PCA Packaging Corporation of America for making the cardboard cubes
- Ohio Dominican University
- St. Paul's Cathedral Archives

Chapter 2: Educational impact

OVER one thousand school children from the surrounding school districts, and many members of the community attended the exhibition. Below are some of my favorite photos of visitors interacting with the exhibits during opening night and on school visits.

Figure 2.1: School field trip.

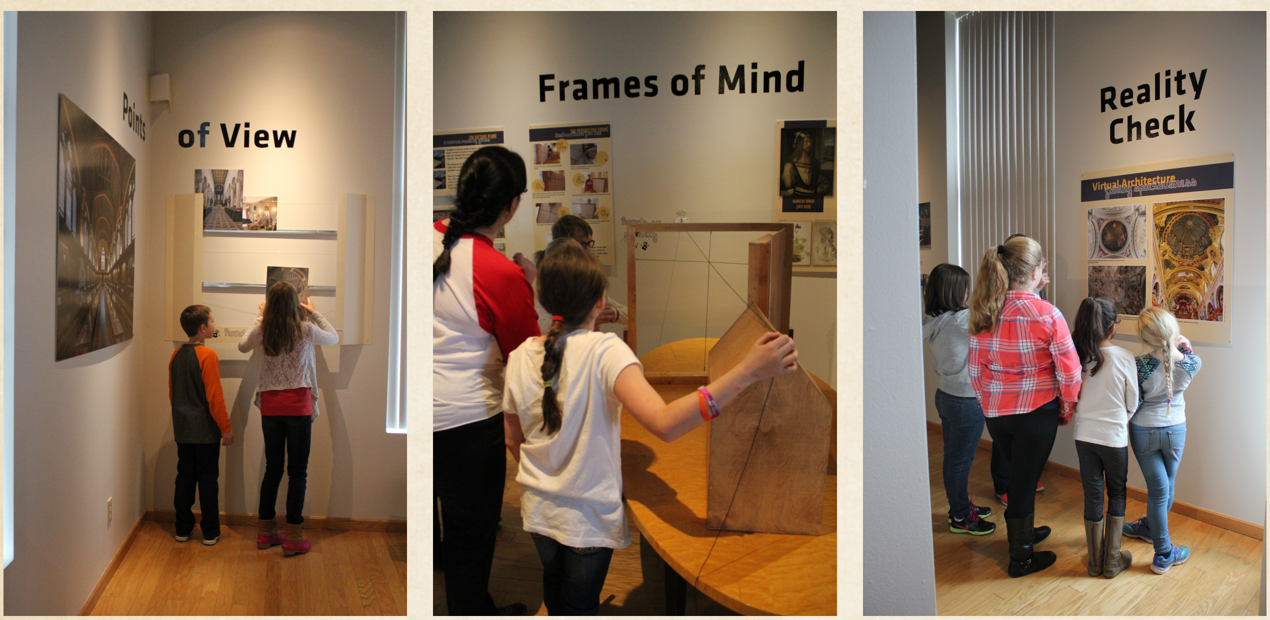


Figure 2.2: Students interacting with hands-on activities (left, center), and looking at fake domes (right).

Figure 2.3: Opening night.

Kid Tech University

Closing day featured a day camp for middle school students called *Kid Tech University*. Activities consisted of an interactive lecture on perspective, anamorphic art stations, and a video about St. Paul's Cathedral.

Perspective Lecture

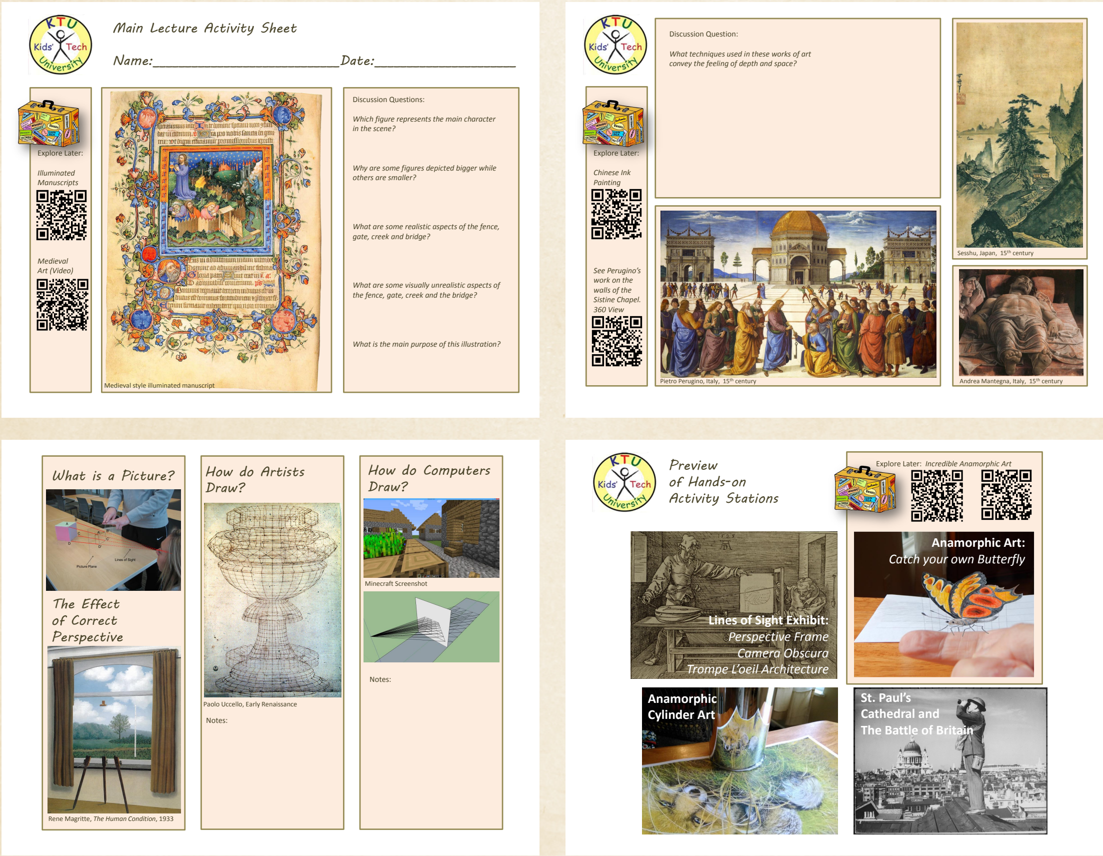


Figure 2.4: The worksheet included activities that students completed during the lecture as well as QR codes to websites they can explore later.

Anamorphic Butterfly Activity Station

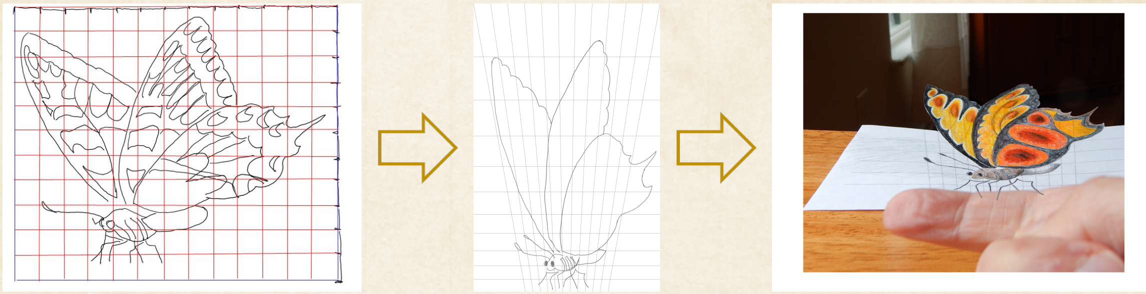


Figure 2.5: Metamorphosis of a butterfly: original sketch, sketch transformed using an anamorphic grid, anamorphic image is colored and cut out.

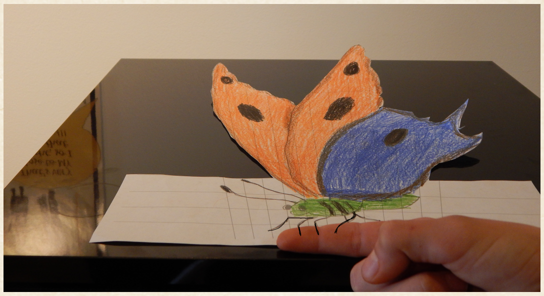


Figure 2.6: Butterfly created by a middle school student.

Anamorphic Cylinder Activity Station

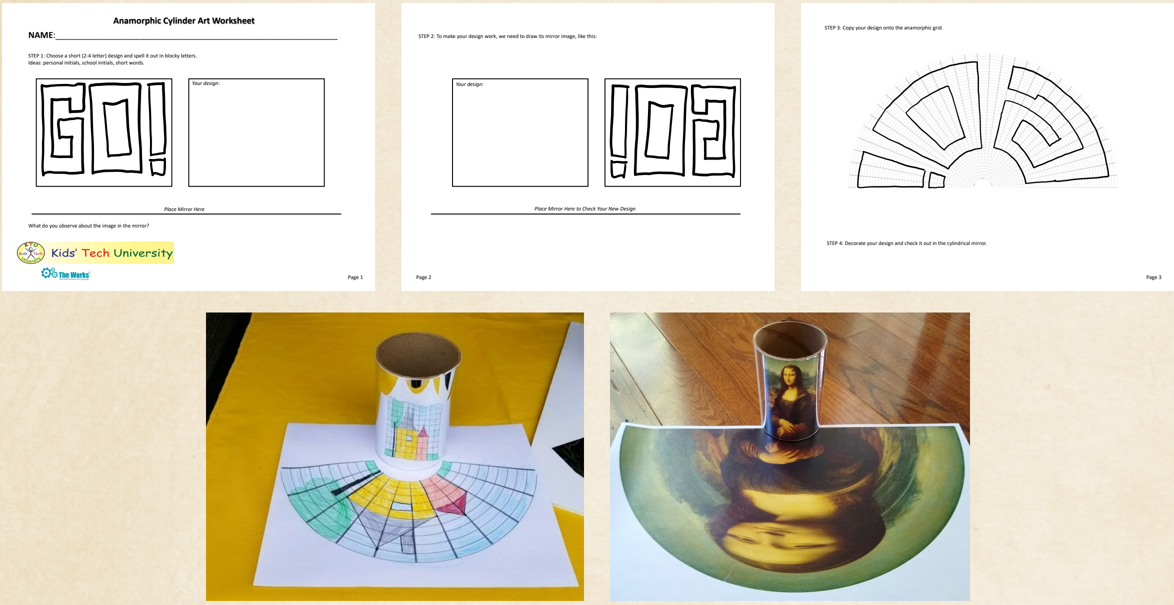


Figure 2.7: Students had an opportunity to create their own anamorphic art and to enjoy pre-made images.

Chapter 3: Taking the show on the road

Following the exhibition at *The Works*, I continued to develop interactive activities and visual displays for use in workshops and public venues. Many of the newer interactives include digital components such as animations, and encourage viewers to use their camera phones. Smaller, updated versions of the *Lines of Sight* exhibition took part in the *COSI Science Festival* and in the *Ohio State Fair*.

COSI Science Festival - May 2019

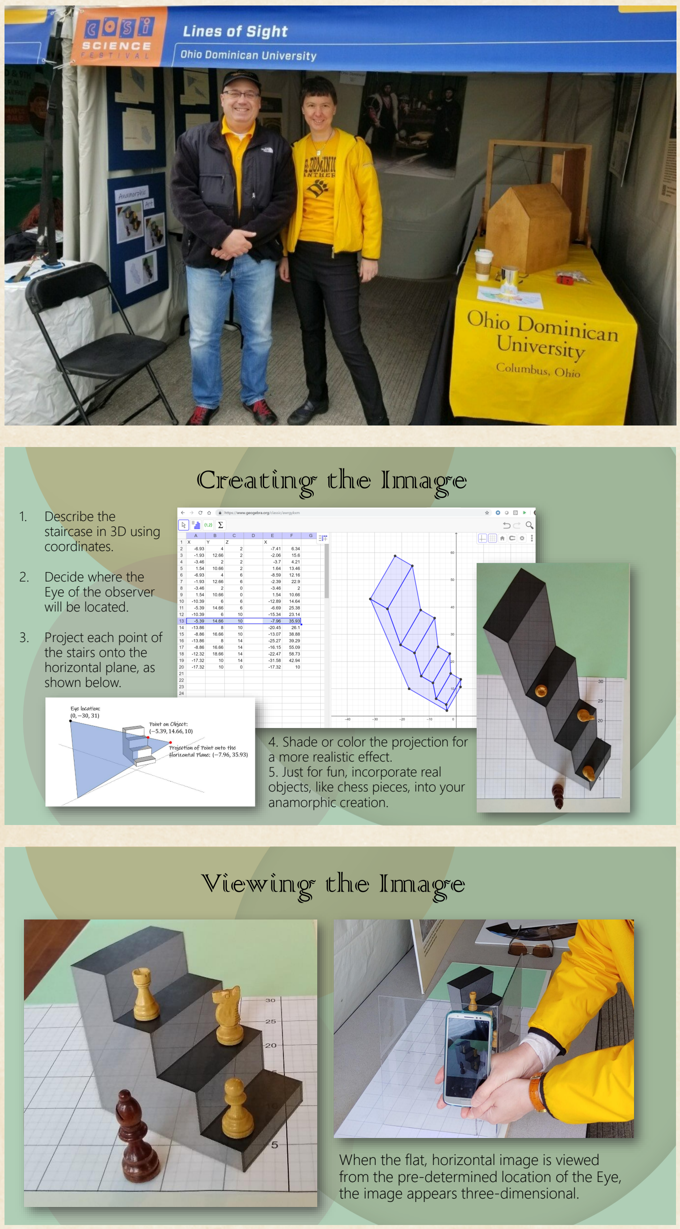


Figure 3.1: This booth exhibition in May of 2019 contained some of the original posters and interactives from the *Lines of Sight* exhibition, and it also featured a newly developed anamorphic staircase. Thanks, Manuel Martinez and Ron Zielke for hosting this with me!

Ohio State Fair - July 2019

While hosting a booth at the *COSI Science Festival*, we were invited by the *Ohio Technology & Engineering Educators Association* to participate in a showcase at the *Ohio State Fair*. Having the showcase take place indoors allowed us to incorporate several digital displays into the show.

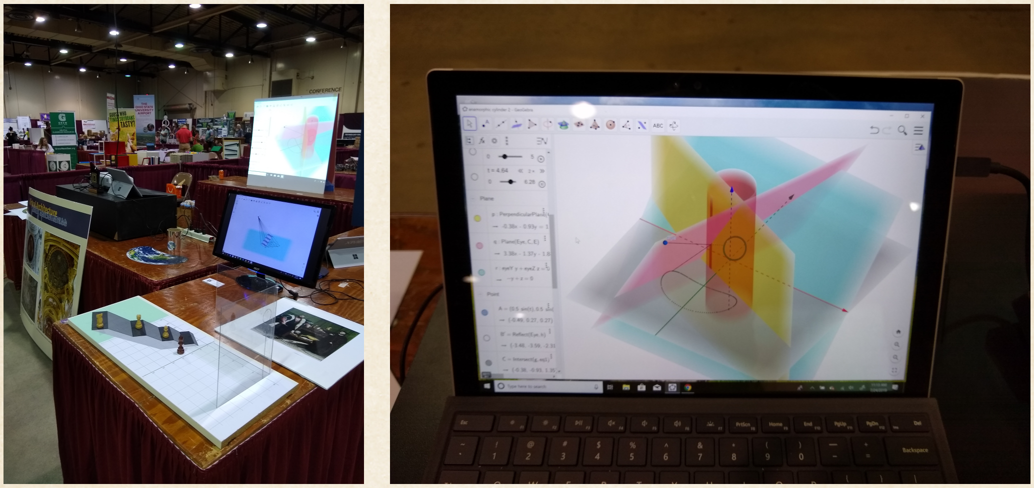
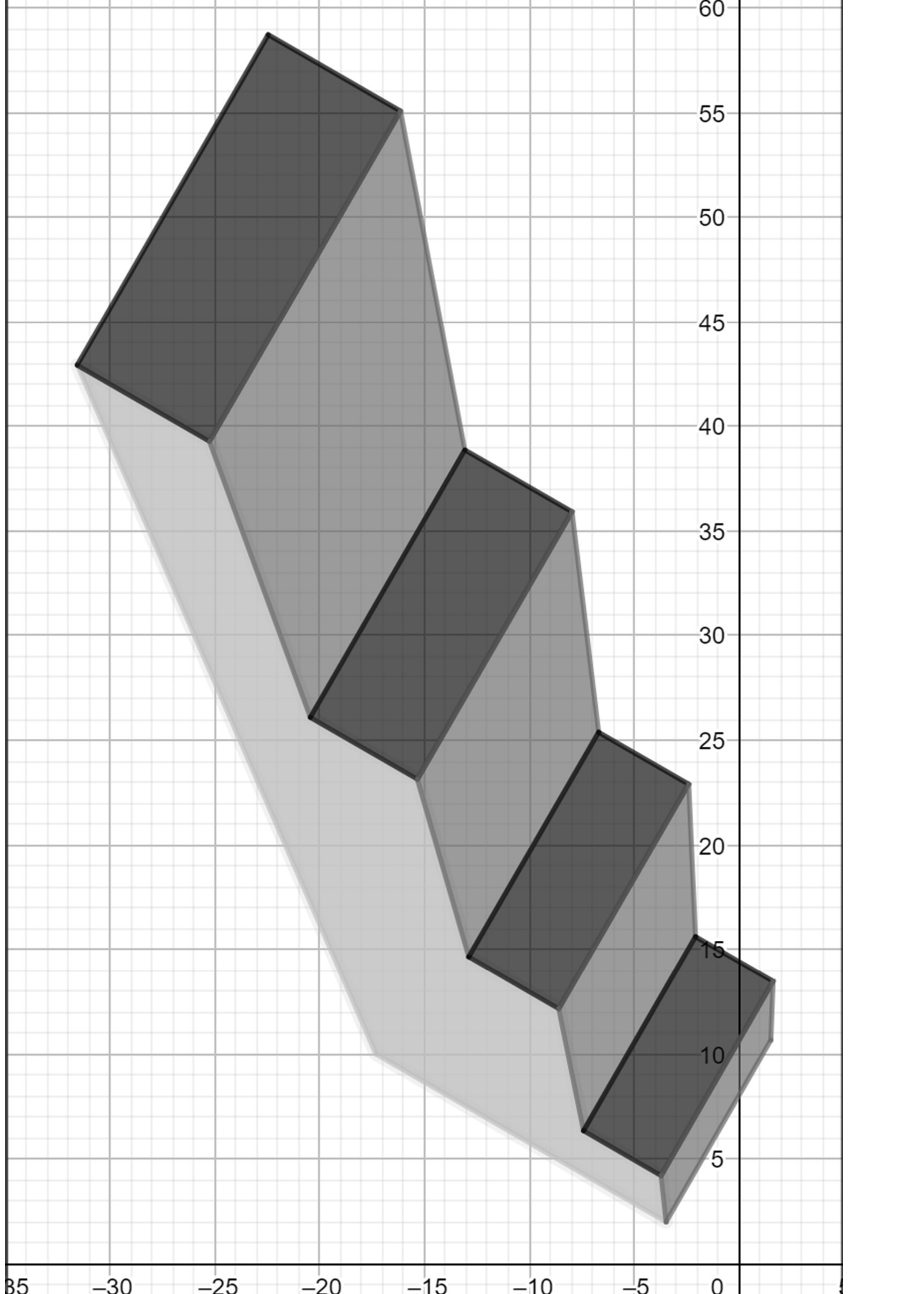


Figure 3.2: Digital animations explain the geometry behind the anamorphic staircase and the anamorphic cylinder.

Figure 3.3: We even got an award!

Acknowledgements

Many thanks to
Manuel Martinez, for organizing the *COSI* booth and hanging out with us in the rain.
Ron Zielke, for always being ready to take part in a new perspective adventure.
Ohio Technology and Engineering Educators Association, for inviting us to participate in the State Fair.



60
55
50
45
40
35
30
25
20
15
10
5
0
−35
−30
−25
−20
−15
−10
−5