



Lines of Sight: Activities Related to Visual Perspective

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Ohio Dominican University

MathFest 2024

Two Activities:

1. Where was Eye?
2. Shady Business

Both can be done together in a two-hour math circle.

Questions?

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Lines of Sight

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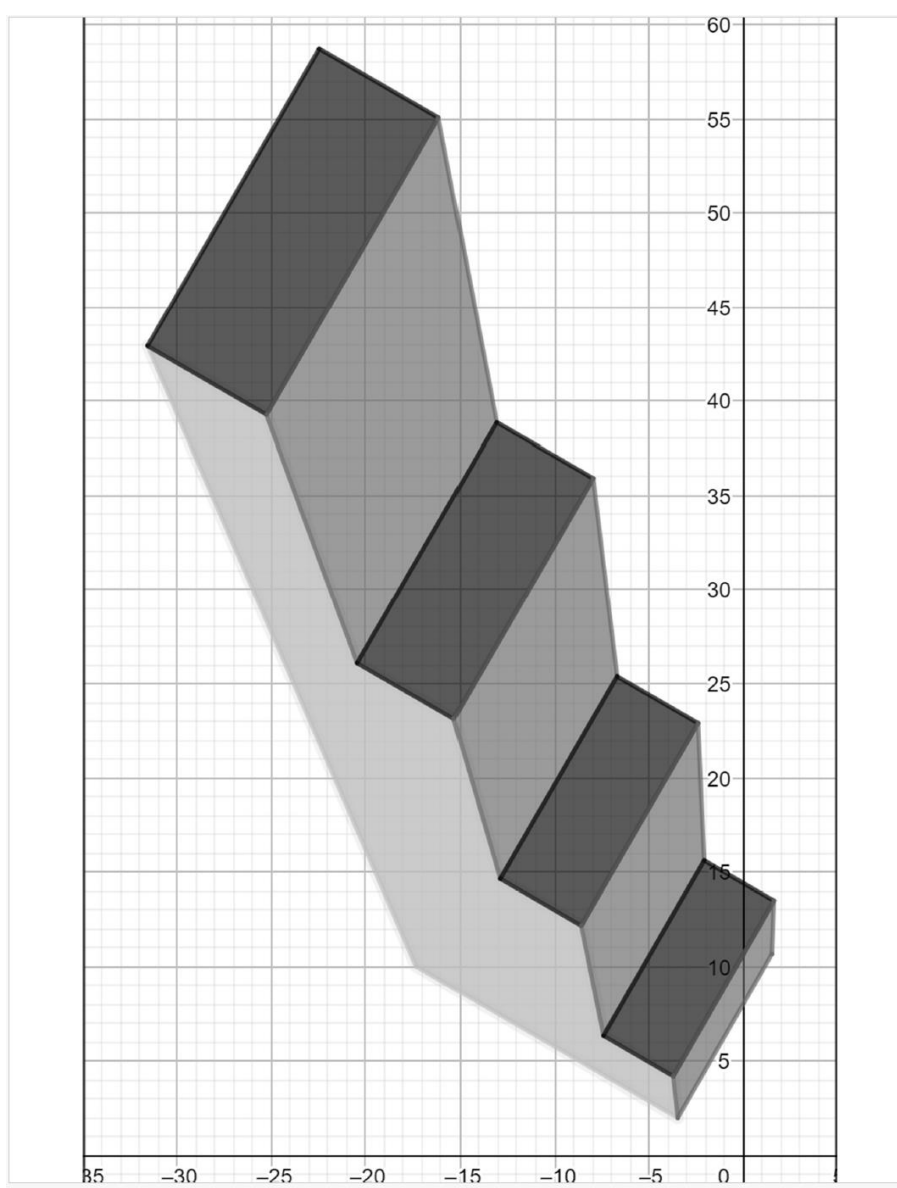
Exhibitor Booth 310



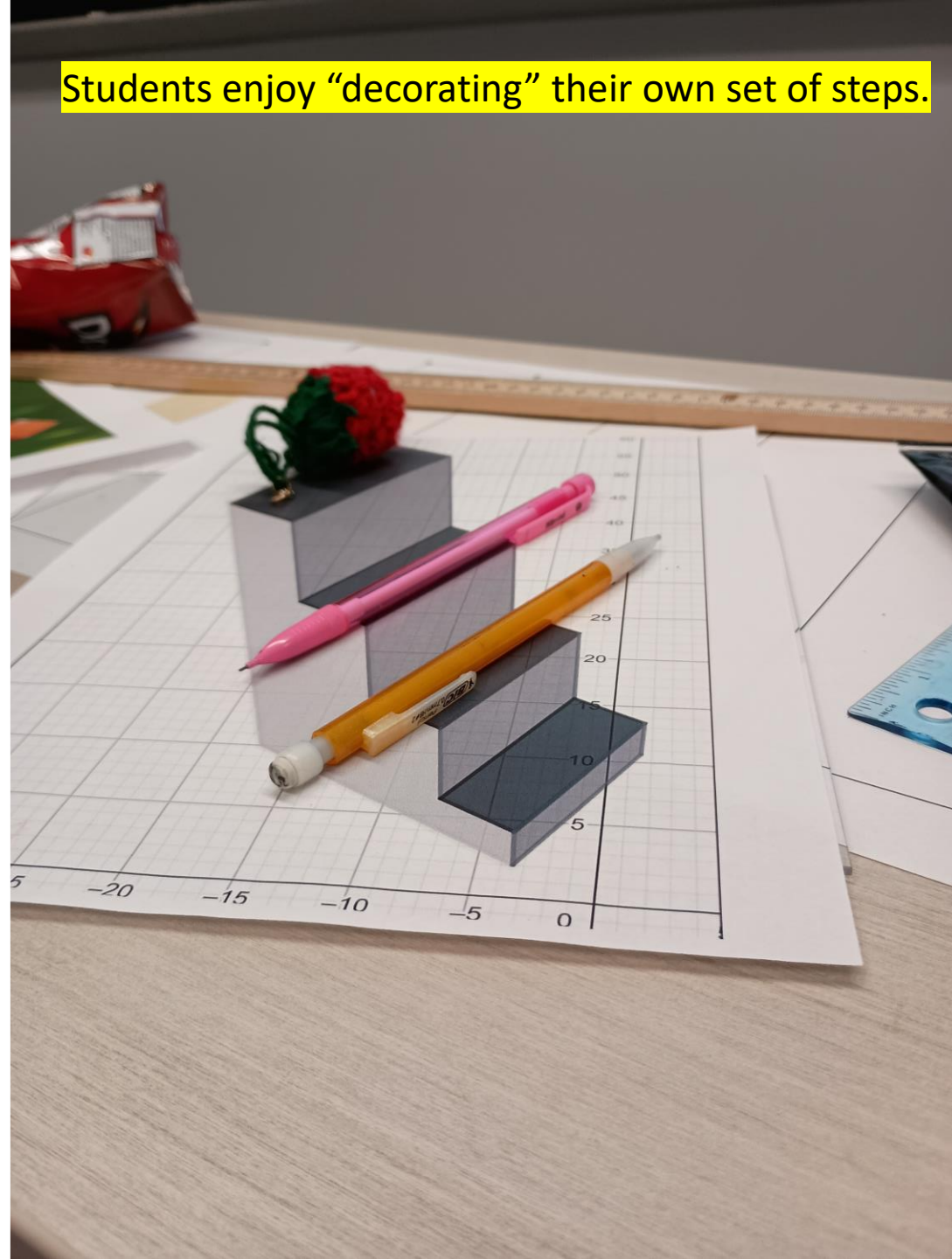
Shady
Business





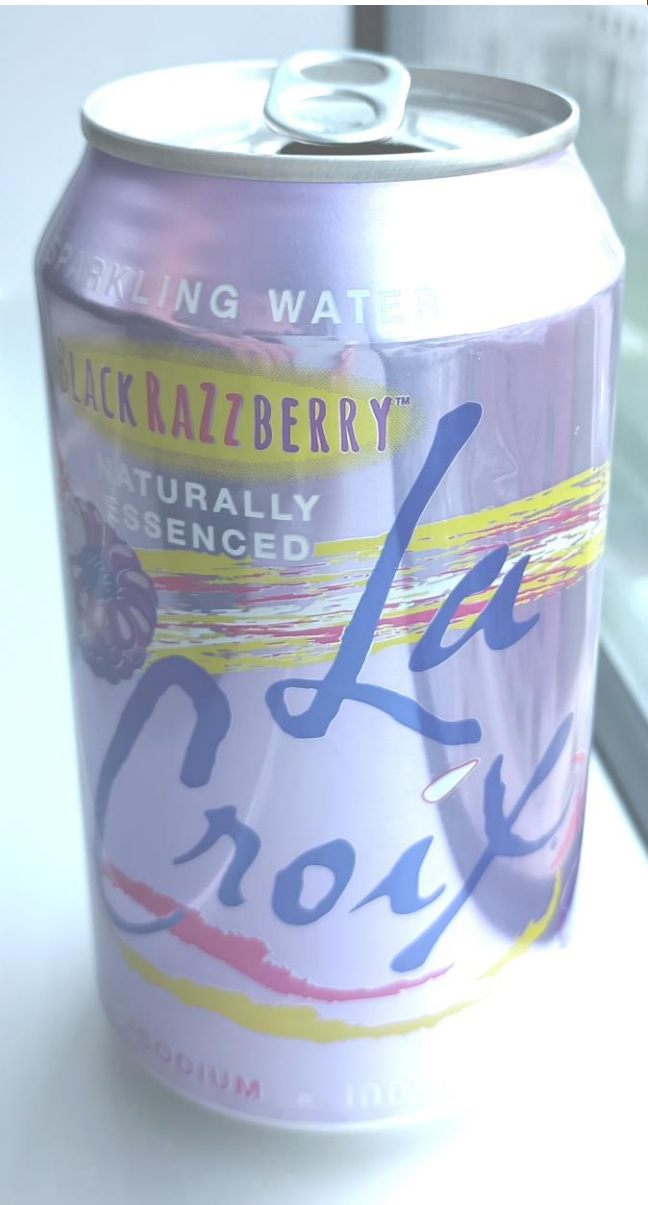


Students enjoy “decorating” their own set of steps.

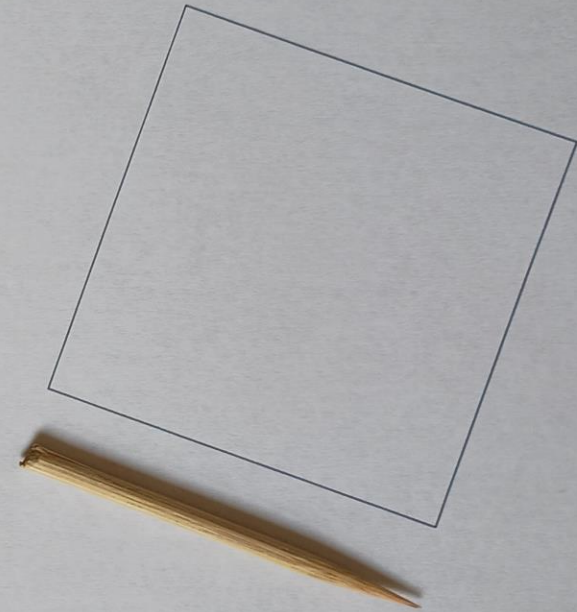


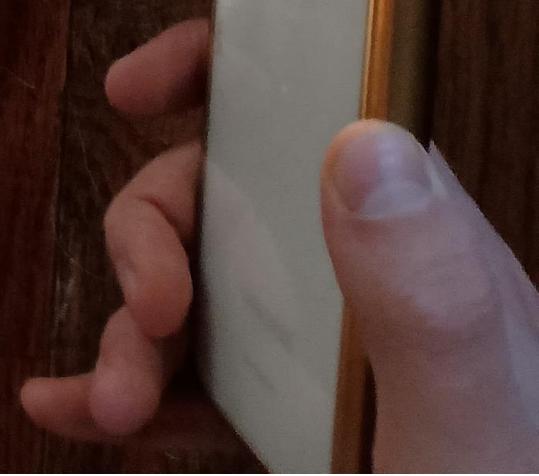
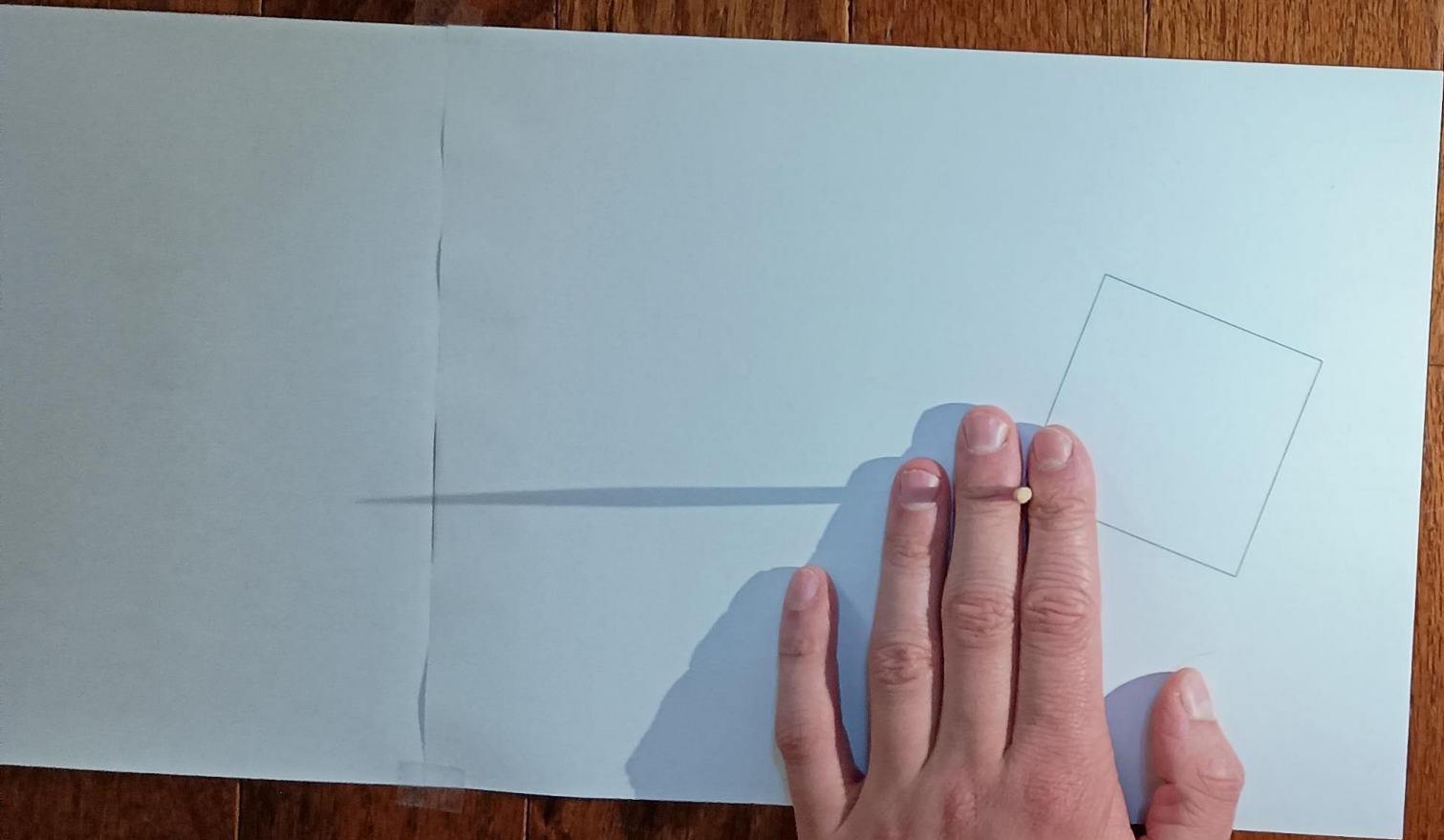
Handout is available at <https://drive.google.com/drive/folders/1mtgz72ZColLCEznaxEnuLPa1-7Yfraw?usp=sharing>

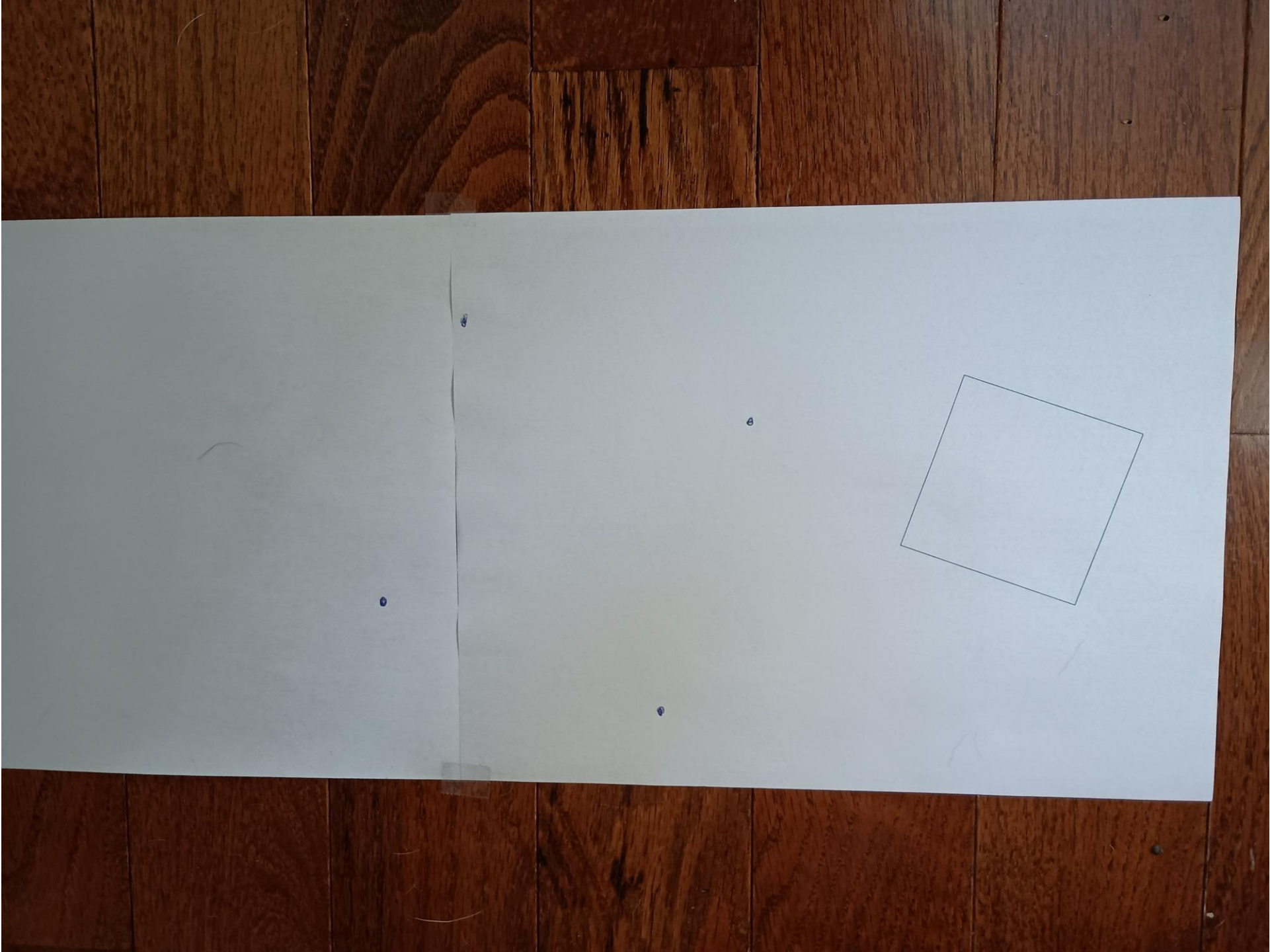
Shady Business:
making anamorphic art with shadows

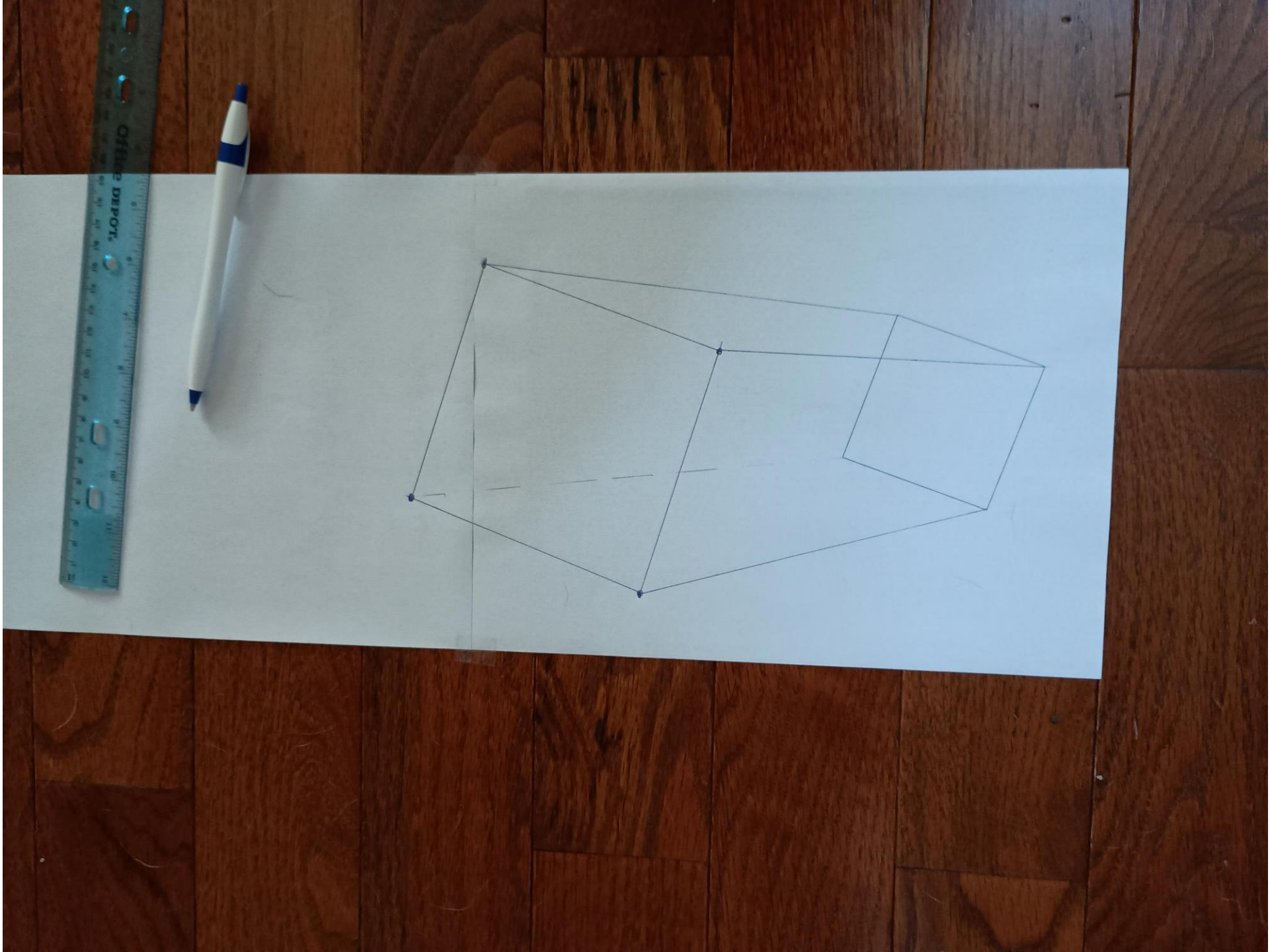


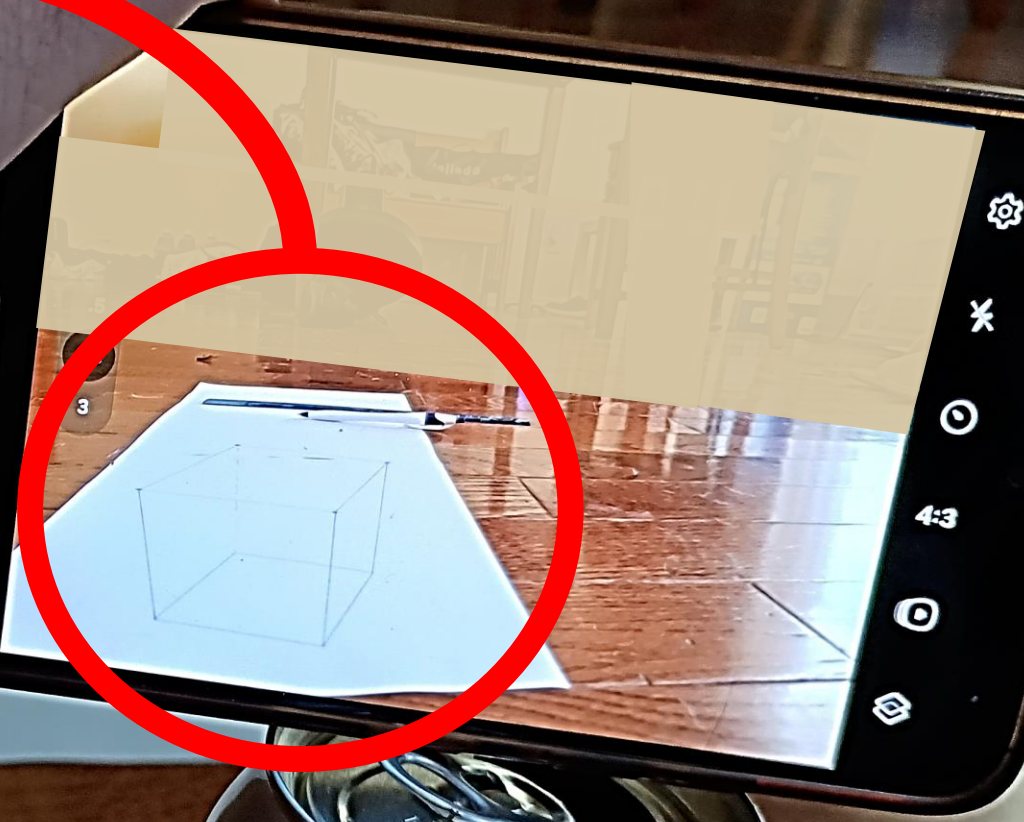
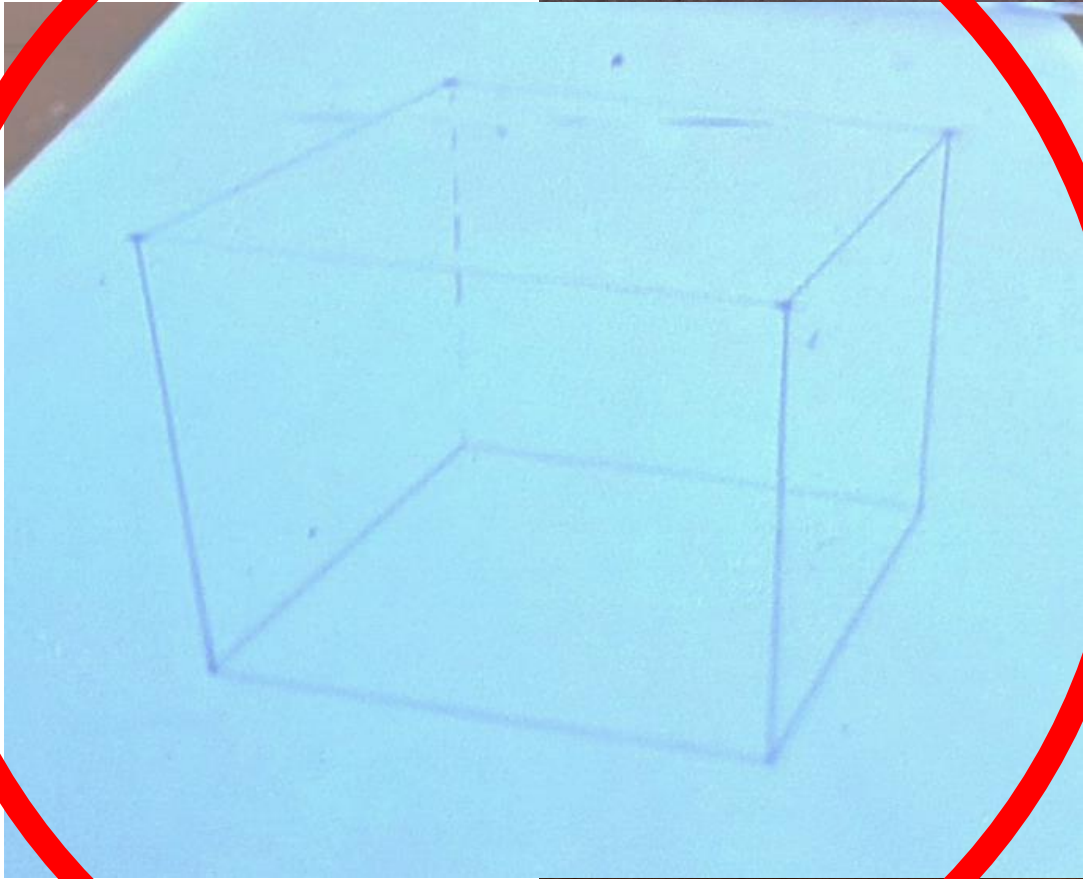
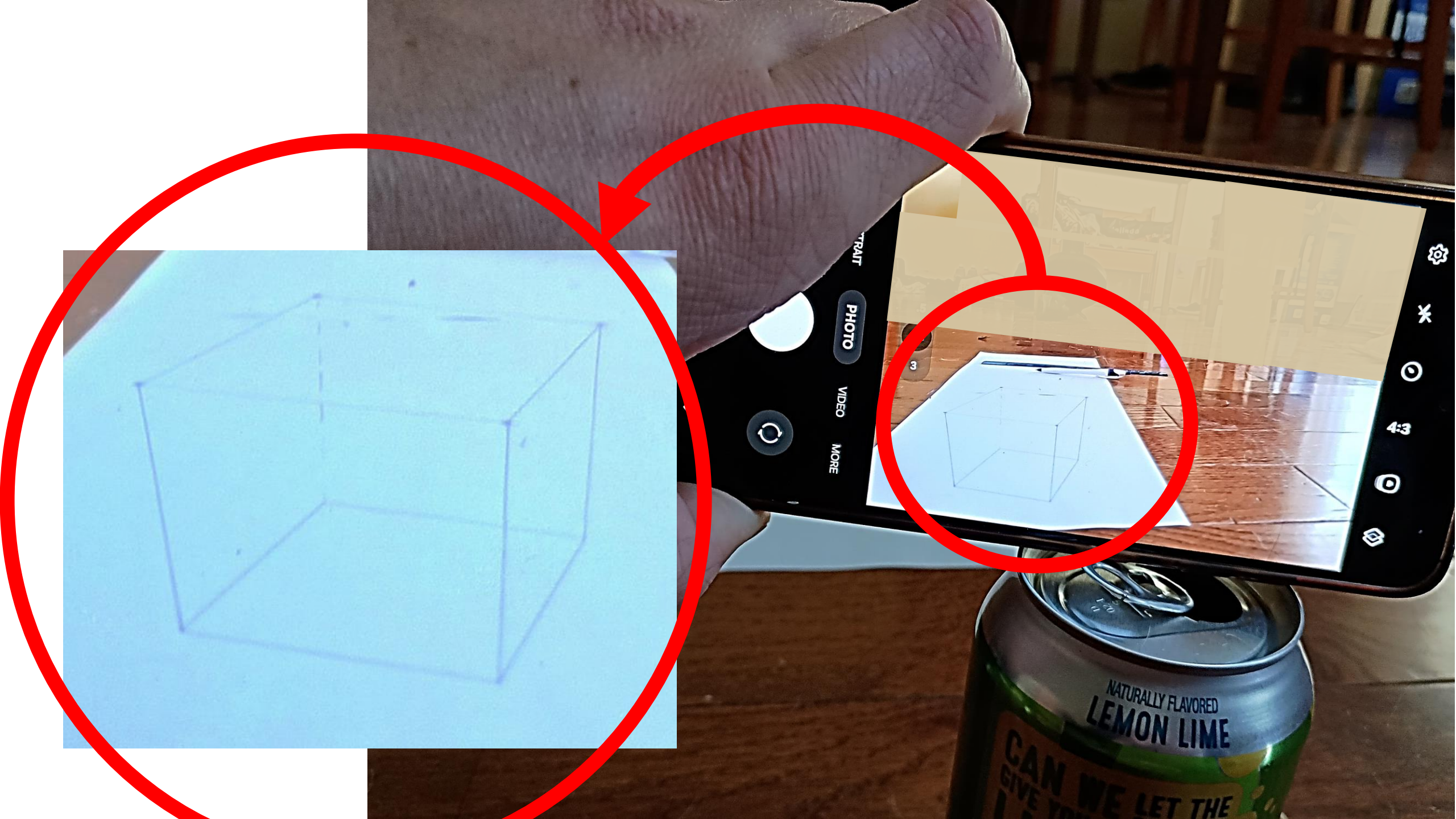
Materials

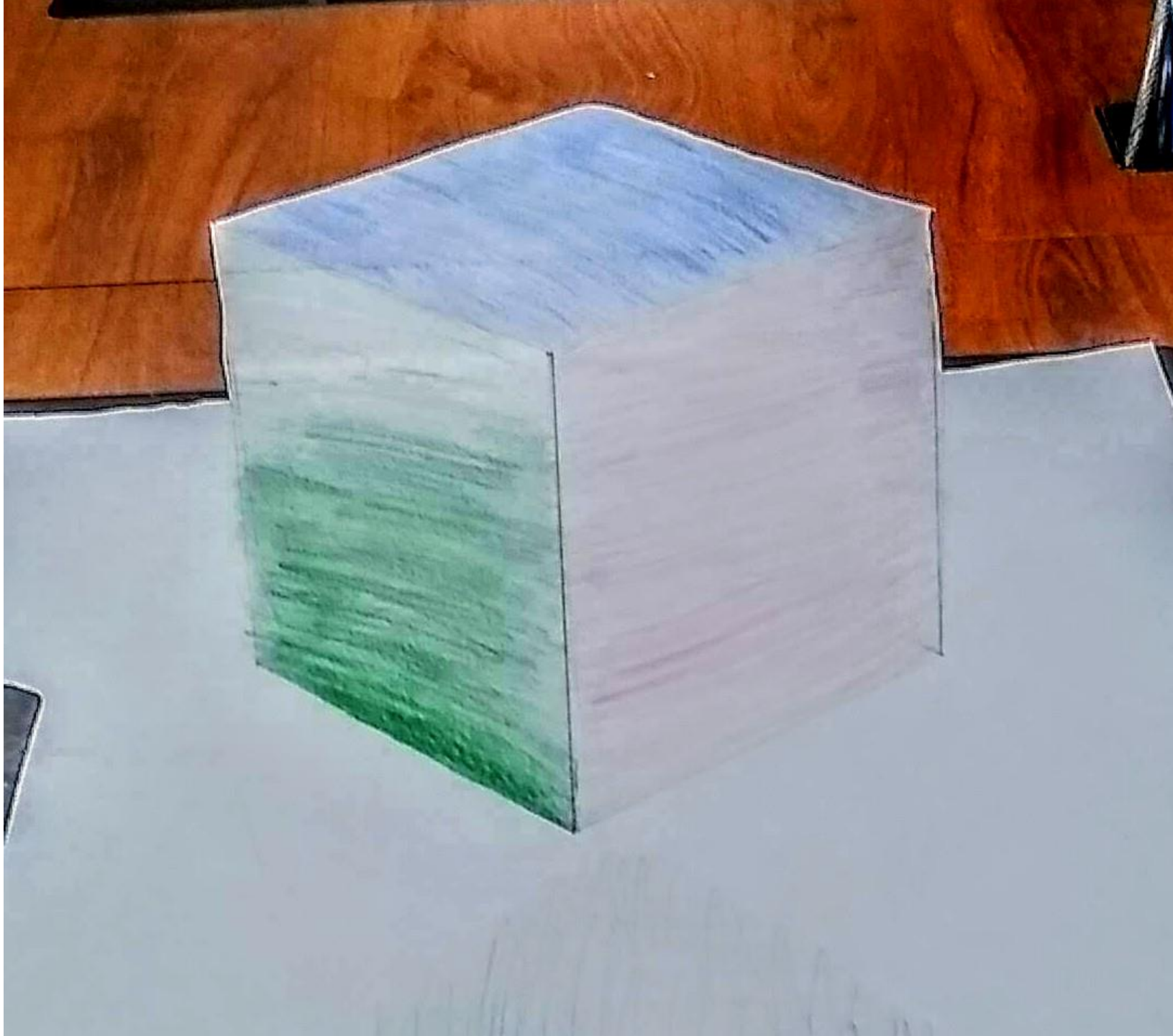














Where was Eye? §1	Where was Eye? (part 1) §1.1	Where was Eye? (part 2) §1.2	Where was Eye? (part 3) §1.3

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Where was Eye? (part 1)

Exploration 1. Three students, Adam, Benjamin, and Cayla, took part in a photography competition. All three submitted photos of railroad tracks. Adam and Benjamin took photos from their natural height; Cayla flew a drone high above her head to take a picture.



(A) Adam

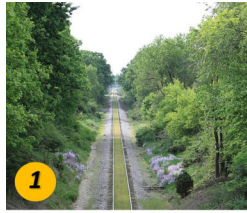


(B) Benjamin



(C) Cayla

The images Adam, Benjamin, and Cayla took appear below (in no particular order).



Match each image with the photographer who took it.

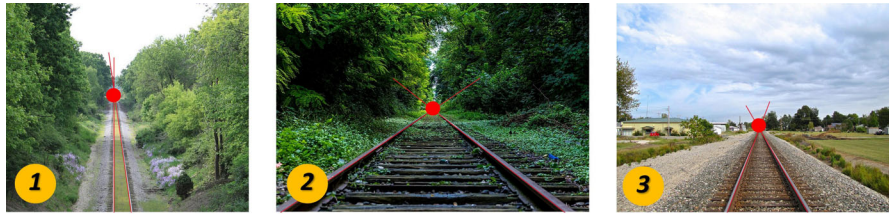
Photo 1 was taken by

Photo 2 was taken by

Photo 3 was taken by

Group Discussion Prompt: *Discuss the reasons for your choices. Do you think it is possible to use these photographs to estimate the height of the camera that took each photo?*

Exploration 2. Let's look at the three photos geometrically.



The rails appear to meet at a single point. Do you know what this point is called?

Center point

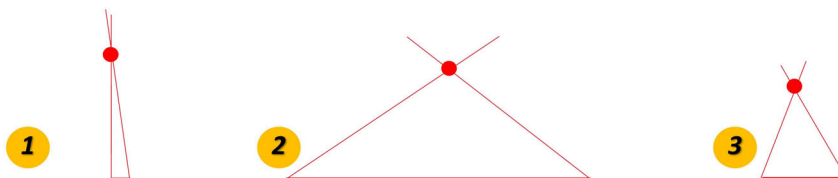
Vanishing point

Vertex

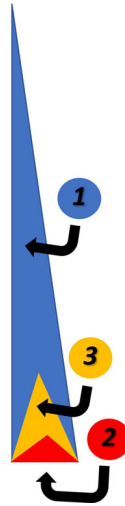
End point

? Check work

The rails, together with the bottom of each photo, form triangles.



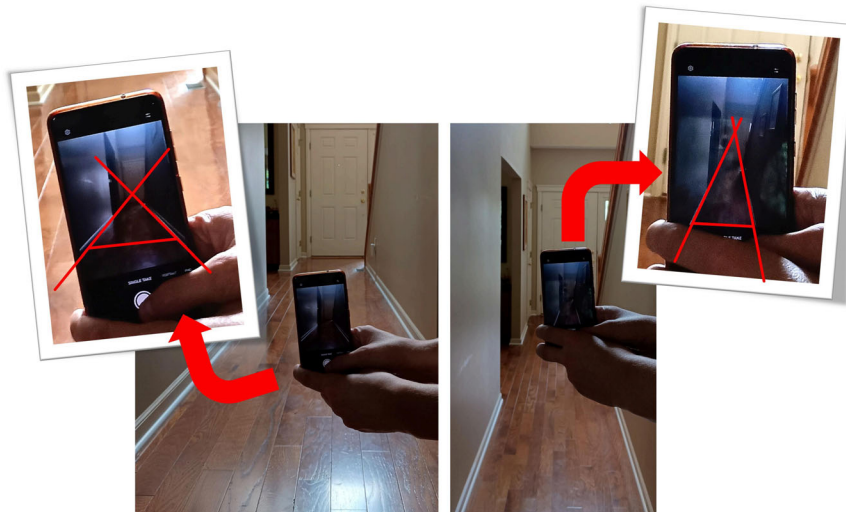
You might describe the first triangle as tall and narrow, and the second triangle as short and wide. These descriptions refer to the *proportions* of these triangles. To really see the difference in the proportions, we can make the bases of the triangles the same size. This makes sense to do because railroad tracks have the same width everywhere in the U.S.



Group Discussion Prompt: *In the previous exploration, you figured out who took what photo. Discuss the relationship between the height of the camera and the height of the triangles. Click on the arrow (below, right) for a hint.*

To see the change in the height of the triangle as the camera height changes, you can use your own phone camera. Find a long hallway or a long table. Position your camera in the middle of the hallway (table). Move the camera up and down observing how the edges of the hallway (table) create taller and shorter triangles, as shown in the photos below.





As the camera lowers, the triangle becomes proportionally shorter.

As the camera lowers, the triangle becomes proportionally taller.

As the camera lowers, the triangle's proportions do not change.

? Check work

So far, we have established that there is a relationship between the proportions of the triangle formed by rails (or edges of a hallway), and the height of the camera that took the photo. Next, we will look at a photo for which the measurements of the real-life hallway, and the height of the camera are known. We will numerically relate the proportions of the triangle to the height of the camera.

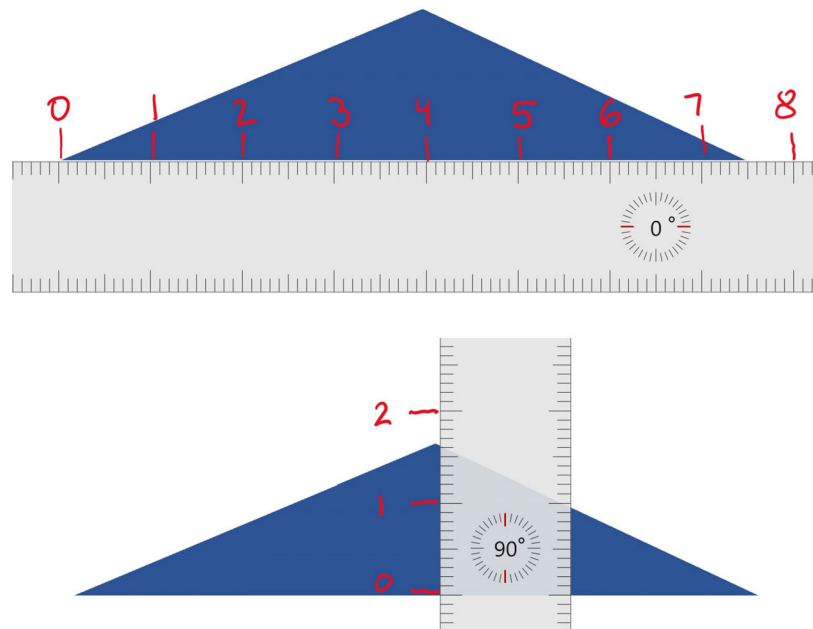
Exploration 3.

The following photo shows a 92-inch wide hallway. The photo was taken with the camera lens 20 inches above the floor.



Observe that the resulting triangle is nearly isosceles. We achieved this by placing the camera as close to the middle of the hallway as we could.

We will now look at the ratio of the height of the triangle to the length of the base.



$$\frac{\text{height}}{\text{base}} = \frac{1.65}{7.5} = 0.22$$

Group Discussion Prompt: *Note that our calculations omitted units. Do units matter in this case? If we were to do our measurements using different units, would the ratio still be the same?*

Next, we will try to figure out what to do with this ratio to find an estimate for the height of the camera. To do this, suppose we made four prints of our photo. The first print is the size of a small poster, the last print is life-sized. The resulting triangles are clearly of different sizes, but they have the same *proportions*! Such triangles are called



Since the last "print" in the above graphic is life-sized, and the width of the real-life hallway is 92 inches, we can conclude that the base of the last triangle is 92 inches. Set up and solve a proportion to find the height of the last blue triangle.

$$\frac{1.65}{7.5} = 0.22 = \frac{\text{height of last triangle (inches)}}{\quad}$$

in

Height of last triangle = in

Compare your answer to the height of the camera given at the start of the problem. The numbers are very close! Do you think this is a coincidence?

Group Discussion Prompt: *Formulate a rule for finding the height of the camera for photos with the same set up. How would you verify that your rule works for ALL such photos? Discuss this with your classmates and your teacher.*

Group Discussion Prompt: *The rule you stated in the previous discussion question is a theoretical rule. In practice, the height of the camera above the floor was given to be 20 inches, but the computed height of the triangle was not exactly 20. What do you think accounts for the*

difference?

Group Discussion Prompt: *Railroads in the U.S. are built so that the distance between the rails is the same everywhere in the country. The distance between the rails is call the gauge. You can learn more about railroad gauges [here](#). Discuss how you can use this information to find the actual heights that Adam, Benjamin, and Cayla took their pictures from.*

In the next two parts of this activity, you will get to perform a photo experiment yourself, and develop the theoretical underpinnings for the rule you discovered here.

Photo Credits

Railroad track photos used in this activity were downloaded from Wikimedia Commons.

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Microsoft clip art was used for student photos.

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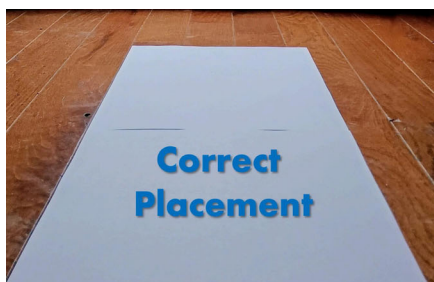
Where was Eye? (part 2)

In this part of the activity, you will take your own photos from a known height to test the formula you had developed in Part 1.

Directions

In groups of two or three, you will use a phone to take photos of a long desk, a sheet of paper or a hallway while holding your phone at a known height.

- Make sure that the camera is located in the center of the hallway/desk/paper so that the triangle formed by the edges is isosceles.

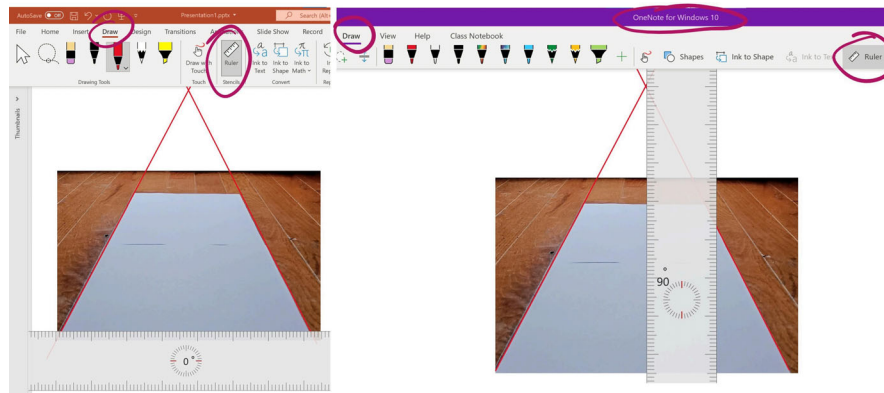


- To keep track of the camera height, hold or tape the phone to a meter stick or a ruler. Have one group member record the vertical distance from the surface (hallway floor/desktop/paper) to the

camera lens for every shot. Remember to measure the distance to the camera lens (not the bottom or the top of the phone).

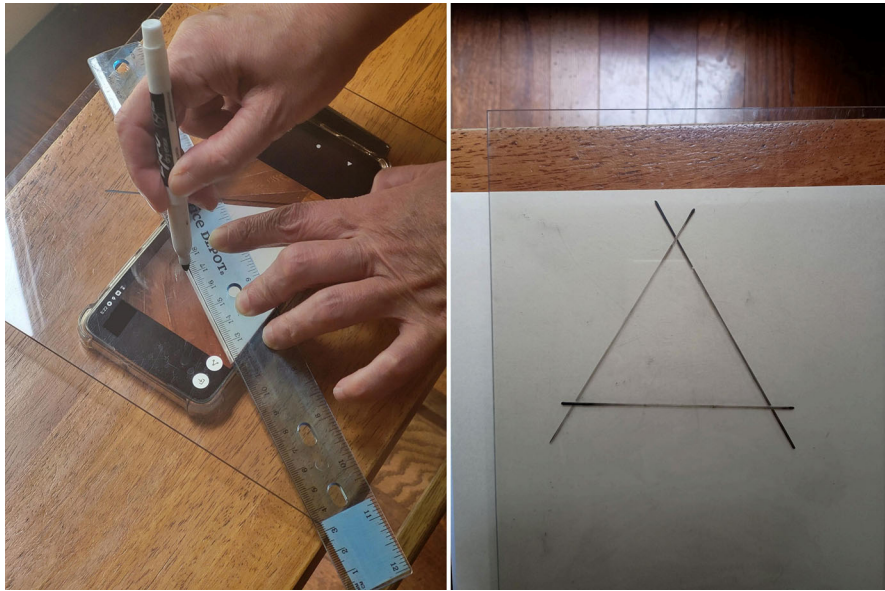
- Below are two methods for drawing and measuring the triangle.

(a) Method 1. If you have a touch-screen computer, import your photos into PowerPoint (left) or OneNote (right). Use the ruler tool (under "Draw") to outline the edges of the hallway/desk/paper in each photo. Form a triangle with the vanishing point as the top vertex, as shown below.



Measure the length of the base, and the height of the triangle and record your measurements.

(b) Method 2. You can draw the triangle directly on your phone photo by overlaying a piece of Plexiglass over your photo and using a dry-erase marker and a ruler to trace and extend the edges of the hallway/desk/paper to form a triangle, as shown below (left).



You can now do your measurements on the tracing (right).

- Follow the procedure you developed in Part 1 to find the height of the camera using ratios. Compare your computed height to your measured height. How close did you get?

Remark. If done carefully, these photo experiments typically produce good results. If your relative error, $\left(\frac{|\text{measured height} - \text{computed height}|}{\text{measured height}}\right)$, is greater than five percent, consider the following sources of error:

- Did you set up your ratios correctly? Did you solve the equation correctly?
- Is your triangle nearly isosceles?
- Did you measure the vertical distance to the actual camera lens? (It is a common mistake

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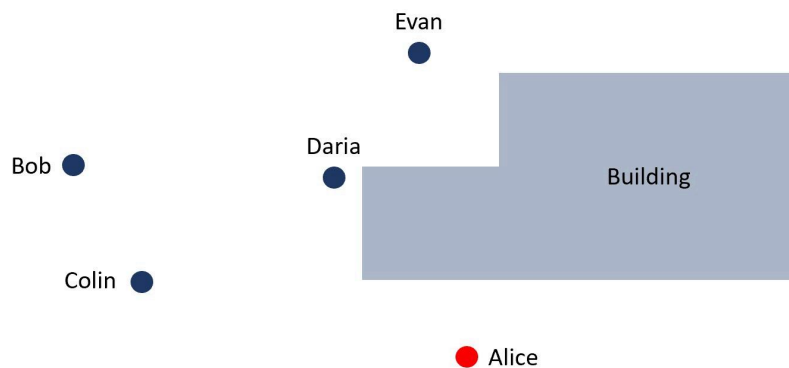
Where was Eye? ^{\$1}	Where was Eye? (part 1) ^{\$1.1}	Where was Eye? (part 2) ^{\$1.2}	Where was Eye? (part 3) ^{\$1.3}

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Where was Eye? (part 3)

We will now develop a theoretical foundation for our method of figuring out the height of the camera.

Exploration 1. Suppose Alice, Bob, Colin, Daria, and Evan are playing hide-and-seek in the school yard. Alice is the seeker. The diagram below shows the location of the players. Which of the children is visible to Alice?



Check the names of all the children that Alice can see.

 Bob

 Colin

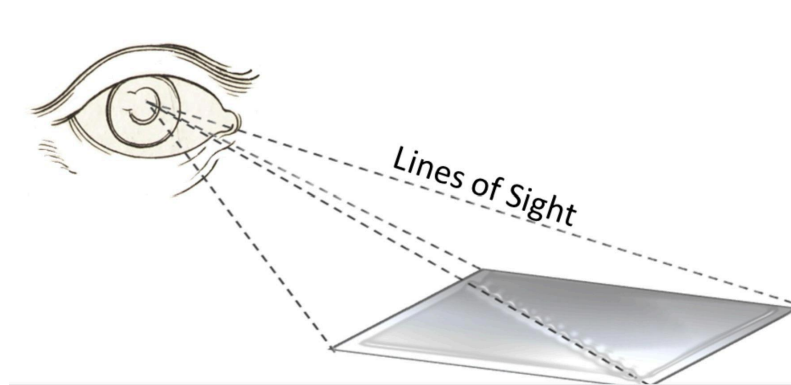
 Daria

 Evan

Group Discussion Prompt: Articulate the reason for your choices. If you need help formulating your thoughts, click on the arrow (below, right), and use the diagram to help you.



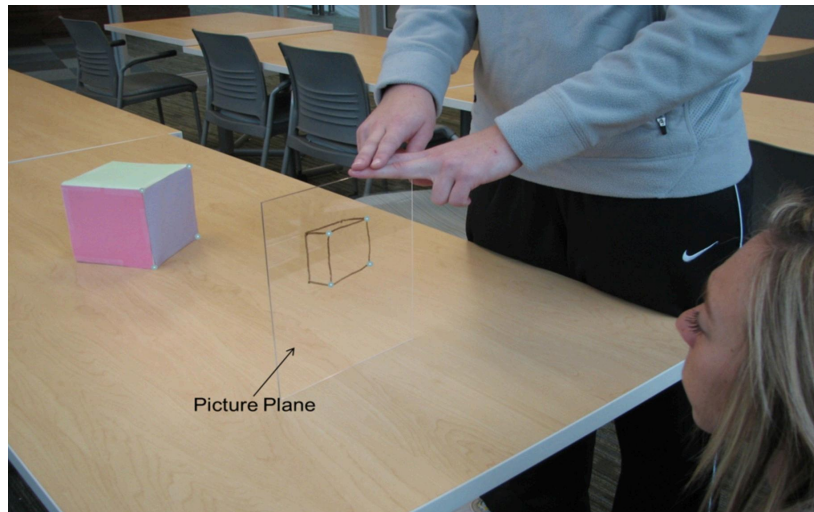
The above exploration intuitively established a very important fact: we see along straight lines. These lines are called *lines of sight*.



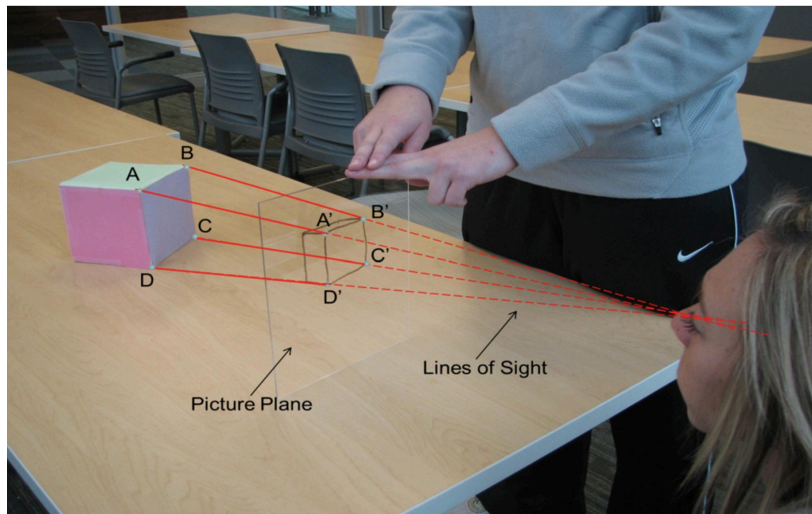
Picture Planes

To understand how lines of sight can help us create a realistic picture, imagine a canvas made of glass. If you position the glass canvas in front of the object you want to draw, you can simply trace the object onto the glass with a marker. We will call the glass canvas a *picture plane*.

Take a look at the photograph below. The student in the photo has just finished tracing the cube onto the glass. From her point of view, the tracing matches up with the cube.



Now let's draw lines of sight that connect the corners of the cube with the eye. The line of sight from each corner of the cube passes through its image on the glass!



This principle allows artists and computer programmers to draw any object from the point of view of an imaginary eye. If we were to place a camera where the student's eye was located, the picture taken by the camera would match the tracing on the glass.

Vanishing Point and the Height of the Eye (Camera)

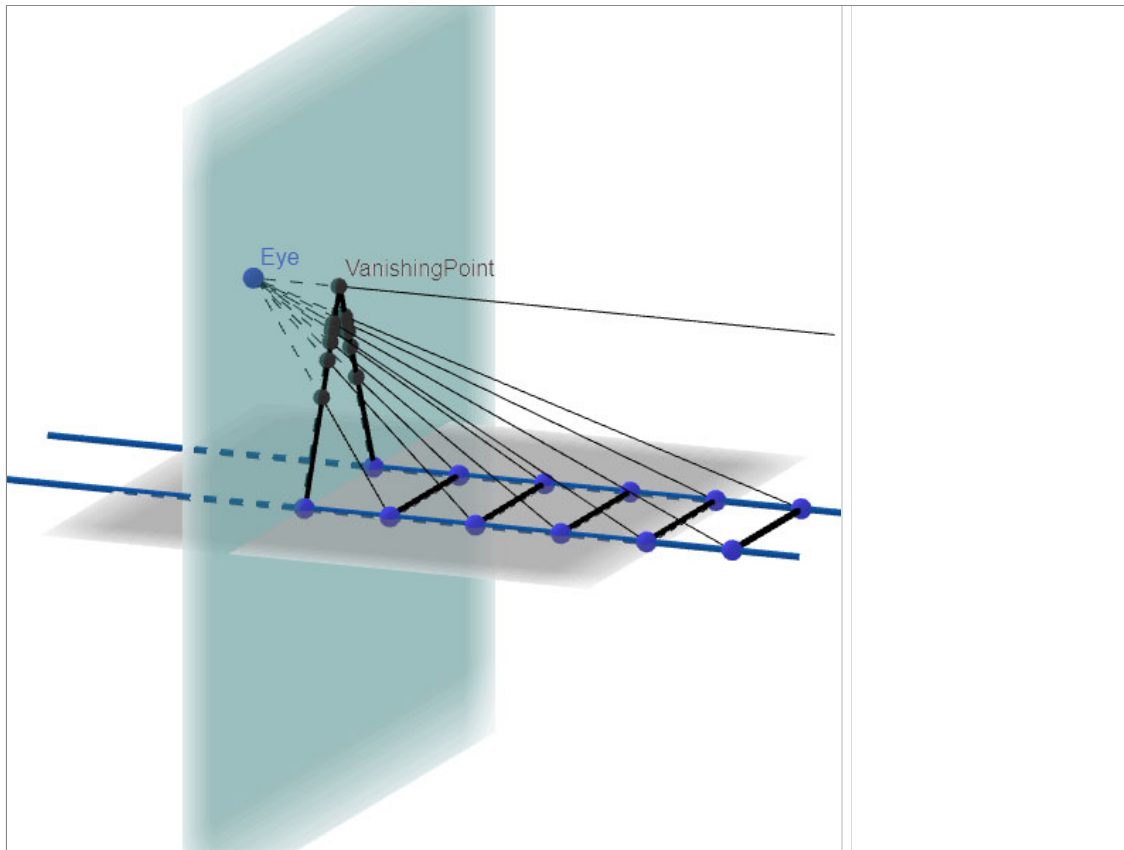
Now that we know about lines of sight and the picture plane, we are ready to figure out why the method we discovered and tested in the first two parts of the activity actually works.

Exploration 2. Recall that when a life-sized version of an image was used, the height of the triangle was equal to the height of the camera.



The interactive model below shows railroad tracks and the Eye (camera) looking at the tracks. The vertical plane is a picture plane. The triangle in the picture plane is the image of the rails the Eye sees. Note the location of the vanishing point. Observe how the sides of the triangle are formed by points of intersection of lines of sight with the picture plane. You can rotate the model for a better view. Use the slider to adjust the height of the Eye and note how the image in the picture plane changes.


RIGHT-CLICK and DRAG to rotate the model for a better view.



Group Discussion Prompt: *Explain why the height of the eye is the height of the triangle.*

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Lines of Sight

an interdisciplinary exhibition by Anna Davis

2016

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CHAPTER 1: THE MAKING OF AN EXHIBITION

THIS scrapbook contains all posters and other artifacts associated with the *Lines of Sight* exhibition at *The Works* in Newark, OH. The exhibition ran from January 29th to April 10th of 2016. *The Works* is a Smithsonian Affiliate Institution. According to the website, “The Works: Ohio Center for History, Art & Technology is an interactive museum for families of all ages.”

The exhibition was designed to provide a hands-on, interactive experience for children and adults who want to explore mathematics of visual perspective, art, computer graphics, history, and our perception of the visual world. The exhibition was sponsored by *Park National Bank*; cardboard was contributed by *PCA Packaging Corporation of America*.



Figure 1.1: *Lines of Sight* concept proposal. SketchUp 3D model by Anna Davis.



Figure 1.2: Final installation.

EXHIBITION STATIONS

The exhibition was broken up into multiple stations, each addressing a narrow area of interest. By traversing the gallery in the clockwise direction, visitors got acquainted with the history, mathematics, and applications of visual perspective. The following table lists the exhibition stations and their descriptions.

Station title	Description
Strings Attached	Discusses the nature of light and early investigations of visual perspective by Leonardo and Desargues.
Frames of Mind	Introduces the concept of a perspective frame. Contains a working replica of Dürer's perspective frame. Discusses computer graphics and how visuals can affect the brain.
Points of View	Contains interactive exercises and a discussion of the importance of viewing location.
Reality Check	Contains examples of early anamorphic art, virtual architecture, and anamorphic street art.
Eye of the Beholder	Contains a working camera obscura and a discussion on the mechanics of the eye.
Hind Sight	A discussion of how photography and mathematics of straight lines can be used to figure out what happened in the past.

All posters were designed by Anna Davis. Images used in the posters are either in the public domain or were used by permission. Photos of the exhibition are courtesy of *The Works*, Anna Davis, and Tom Brockman.

STATION 1: STRINGS ATTACHED

The aesthetic of this three-dimensional display was inspired by illustrations of Desargues' manuscript on geometry of visual perspective. In Desargues' treatise, lines of sight were depicted as strings, as seen in the posters below.

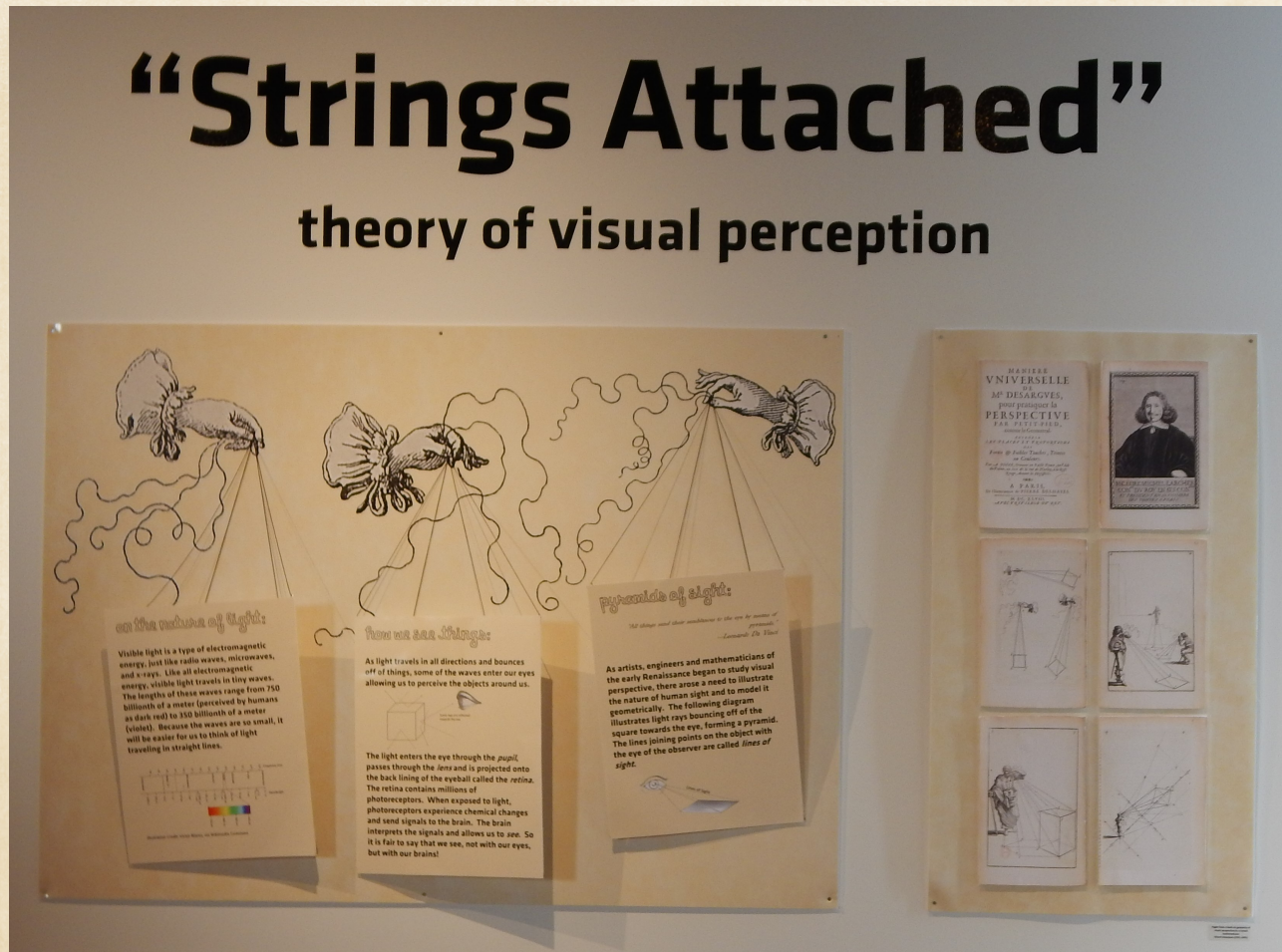


Figure 1.3: *Strings Attached* display lays the geometric foundation of the theory of perspective.

STATION 2: FRAMES OF MIND

This section introduces the concept of a picture plane by considering the use of various types of perspective frames throughout history. A functional replica of Dürer's perspective frame was created by volunteers Col. Bill Snider, Deb Tung and Harvey Tung.

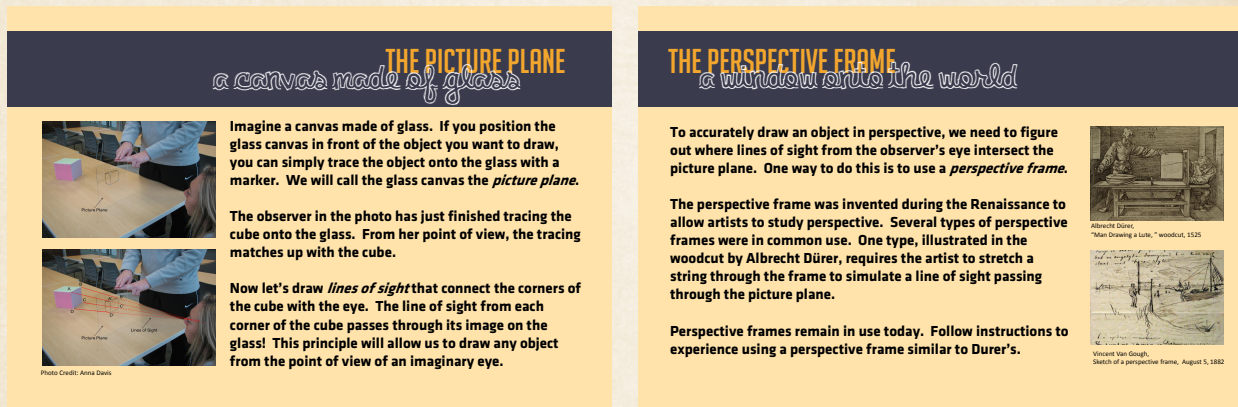


Figure 1.4: The poster on the left features ODU math students using plexiglass as a picture plane to illustrate that lines of sight, which connect points on the object with the corresponding points on the tracing, converge at the eye. The poster on the right shows two perspective frames: one used by Dürer, and one used by Van Gough.



Figure 1.5: A working replica of Dürer's perspective frame was built by Col. Bill Snider, Deb Tung, Harvey Tung, and Anna Davis. The poster on the left provides step-by-step directions for how to use the frame to draw the wooden house. A copy of Dürer's woodcut which inspired this contraption can be seen behind the frame in the middle photo and in Figure 1.4.



Self-Portrait with Landscape, 1488

ALBRECHT DÜRER
[1471-1528]

Albrecht Dürer is generally thought to be the first German artist to understand and utilize techniques developed by the Italians during the early Renaissance. He studied geometry of visual perspective and proportion, and understood the significance of anatomical knowledge in portraying the human body. At the same time, Dürer remained true to many traditions of northern European art, such as exquisite realism and attention to detail.



View of Kalchreut, 1511

Flowers, 1526



Self-Portrait at the Age of Thirteen, 1484



Saint Jerome in His Study, 1514

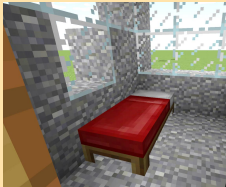


Portrait of Jacob Muffel, 1526



Figure 1.6: Albrecht Dürer and his art.

Next, we use the concept of the picture plane to develop the mathematics behind computer graphics.



Games such as Minecraft® use mathematical algorithms to generate views of 3-dimensional environments in real time, as the user navigates the landscape. Although the code used to generate such landscapes can be highly sophisticated to ensure computational efficiency - we don't want to wait an hour for the cows to come home - the basic principles behind such graphics are the same as the principles behind Dürer's perspective frame. The computer has to figure out where lines of sight, traveling from the user's eye to an imaginary object behind the screen, intersect the picture plane (the screen).

Screenshot Credit: Erik Davis

Disclaimer: NOT AN OFFICIAL MINECRAFT PRODUCT NOT APPROVED BY OR ASSOCIATED WITH MOJANG. Statements made regarding computer code are based on general information about computer graphics and are not meant to provide specific information about the code used by MOJANG.

Images on the Screen perspective projections

To generate an accurate depiction of a three-dimensional object on the computer screen, you need to do the following:

1. Regard the screen as a perspective frame.
2. Design your imaginary 3D object on the other side of the screen, and know the exact coordinates of each point.
3. Make a reasonable guess about the location of the user's eye.
4. Use mathematics to model *lines of sight* connecting the eye with points on your 3D object.
5. Find where lines of sight intersect the screen to create the *image* of the object on the screen.

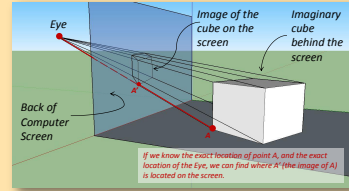


Diagram 1. Perspective Projection of an imaginary cube onto the screen. This model was generated using SketchUp. You can download SketchUp for free at <http://www.sketchup.com>

The process of generating an image in this way is called *perspective projection*. In general, a projection makes an image of a three-dimensional object on a plane. Perspective projection is just one of many kinds of projections. Another example of projections at work is a shadow. A shadow is a flat image produced when light shines on a three-dimensional object.

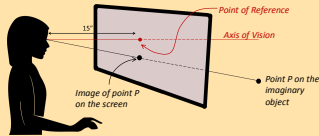


Diagram 2. Shadows are images of projections. Be sure to visit the Look Lab on the first floor for some shadow fun!

How does the computer know?

Part 1

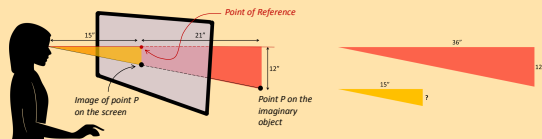
Suppose you are designing an algorithm that would allow viewers to see a specific three-dimensional object on their screens. How would you tell the computer where to plot each point of the object on the screen?



First, let's assume that the user is located 15" away from the screen. We will establish a fixed reference line called the *axis of vision*. This line emanates from the user's eye and is perpendicular to the screen. The point where the axis of vision intersects the screen will be called *point of reference*.

The user will be looking at hundreds of points on the imaginary object. So, there will be hundreds of *lines of sight* connecting her eye to the points on the object. While lines of sight vary, depending on the location of each point on the object, the point of reference will remain fixed. All points on the screen will be plotted relative to the point of reference.

To make things easy, to start with, let's assume that point P on the imaginary object is located 21" away from the screen and 12" directly below the axis of vision. This setup gives us two triangles: a big red triangle, and a smaller yellow triangle. These triangles are called *similar*, because they have the same *proportions*.

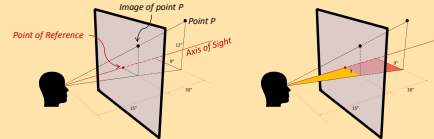


To find the length of the short side of the yellow triangle we look at the proportion: $12/36 = ?/15$. This tells us that the image of point P is located 5" below the point of reference. Now we know where to plot the point!

How does the computer know?

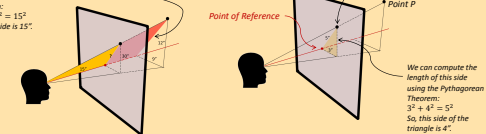
Part 2

What if the point on the imaginary object is NOT located directly above or below the axis of sight?



Suppose that point P is 30" behind the screen, 9" to the right and 12" above the axis of sight. We will assume that the user's eyes are 15" from the screen.

We can compute the length of this side using the Pythagorean Theorem:
 $9^2 + 12^2 = 15^2$
So, this side is 15".



These two triangles are also similar. We can use the Pythagorean Theorem to find one of the sides, as shown in the diagram. Then we find the unknown side of the yellow triangle by looking at the ratio:
 $15/45 = ?/15$
The unknown side is 5".

The two triangles in the diagram are similar. We can find the unknown side by looking at the ratio:
 $9/45 = ?/15$
The unknown side is 3".

We can compute the length of this side using the Pythagorean Theorem:
 $3^2 + 4^2 = 5^2$
So, this side of the triangle is 4".

We use the Pythagorean Theorem, as shown above to find the vertical distance to the image. Now we know that the image is located 3" to the right and 4" up from the reference point. This gives us the exact coordinates where the image should be plotted on the screen.

Figure 1.7: Using familiar virtual environments such as a Minecraft, this set of posters sets the scene for the calculations involved in generating computer graphics.

STATION 3: POINTS OF VIEW

In this section we start to explore how the viewer's vantage point affects the viewer's perception of the image.

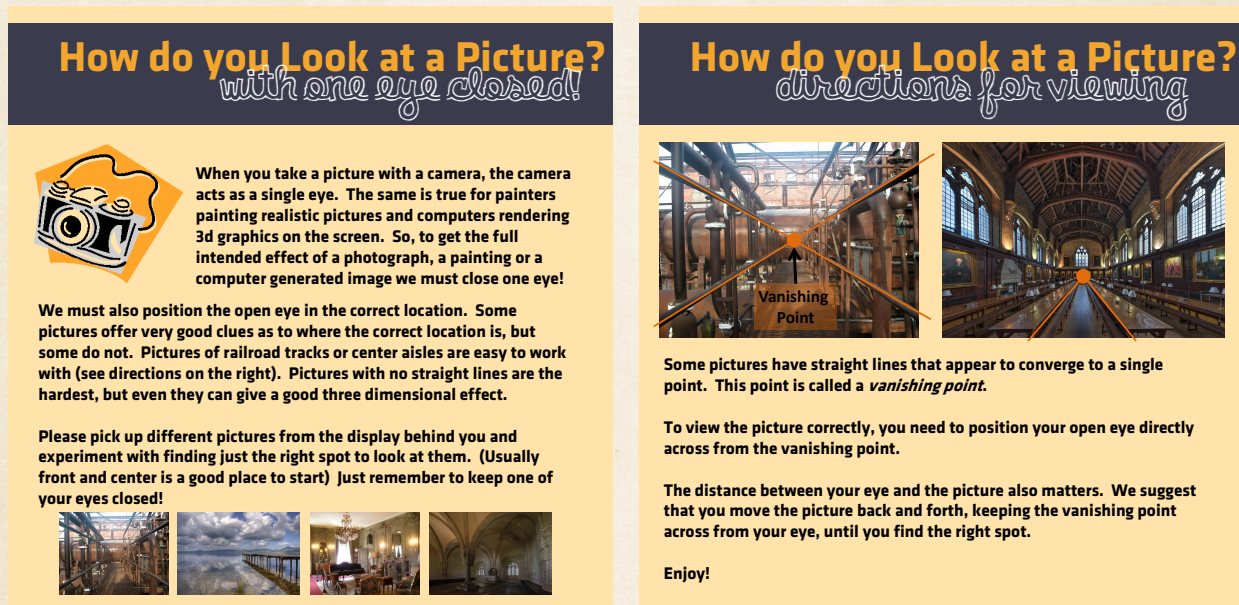


Figure 1.8: These posters discuss how one should look at an image to experience the full perspective effect.




Figure 1.9: A large poster of Balliol College, and multiple smaller prints allow visitors to practice locating the vanishing point and using it to find the perfect vantage point from which to experience each print.

We wrap up this section by looking at how simulated environments can affect the brain.

Visually Induced Motion Sickness *is your screen making you dizzy?*


In 1895, *Psychological Review* published a description by R.W. Wood of his experience with the Fair attraction called the *Haunted Swing*.




"We took our seats and the swing was put in motion, the arc gradually increasing in amplitude until each oscillation carried us apparently into the upper corners of the room. Each vibration of the swing caused those peculiar 'empty' sensations within which one feels in an elevator; and as we rushed backwards towards the top of the room there was a distinct feeling of 'leaning forward,' if I can describe it - such as one always experiences in a backward swing, and an involuntary clutching at the seats to keep from being pitched out. We were then told to hold tightly as the swing was going clear over, and sure enough, so it did..." (Wood, 1895)

Although Wood *experienced* the swing going "clear over" he knew that the swing remained completely stationary throughout his experience; it was the ROOM that rotated around the swing!

The Haunted Swing is a 19th century precursor to modern day virtual reality environments. In such environments a stationary observer is made to experience self-motion by observing moving images. Correct perspective representation of objects is a vital component of making the experience believable. In addition to entertainment, virtual reality environments are used for training. For example, flight simulators provide a safe, realistic alternative to flying.



Unfortunately, fooling the brain into thinking that one's stationary body is in motion can cause ill effects. Sufficient exposure may produce dizziness and nausea. It is believed that the underlying cause of these symptoms is the same as the cause of regular motion sickness which results from a contradictory or an unexpected pattern of sensory signals. In other words, if the eyes are sending a message of motion, while other senses perceive lack of movement, the body may become confused, causing the undesirable symptoms. In addition to being unpleasant, these symptoms can reduce productivity in the workplace where virtual environments are used. Visually Induced Motion Sickness (VIMS) is an area of active research. (Diels, 2008)



hitting the open road

Some visitors to The Works have reported symptoms of Visually Induced Motion Sickness (VIMS) after using the driving simulator. Drive safely!




Figure 1.10: These posters explore historical and modern environment simulators and their potential effect on the brain.

STATION 4: REALITY CHECK

At this station viewers explore how artists can intentionally create images that require a certain (unusual) vantage point to be viewed correctly. The resulting pieces are classified as *anamorphic art*. Throughout history, anamorphic art was used to create hidden images, generate fun effects, and to enhance architectural features.



Figure 1.11: Hans Holbein's famous piece *The Ambassadors* contains a distorted image of a skull in the foreground. When viewed from an extreme angle from the side - as the boy is doing in the photo - the skull appears to pop out of the canvas.



A sketch for the portrait of Thomas More and his family.

Upon the receipt of Holbein's sketch of Thomas More and his family, Erasmus wrote to Thomas More, "... it is so completely successful that I should scarcely be able to see you better if I were with you."



Portrait of Erasmus, c 1523



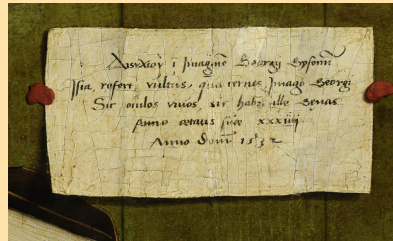
Self-portrait, c.1523

Hans Holbein was born in Augsburg. His father, Hans Holbein the Elder was one of the most prominent painters of his generation. Hans Holbein traveled to Italy and is thought to have been heavily influenced by Leonardo and other Renaissance masters. Hans Holbein's career was primarily divided between Basle, where he attracted the patronage of rich merchants, and England, where he became the court painter to Henry VIII.

**HANS HOLBEIN
THE YOUNGER
(1497-1543)**



Portrait of Georg Giese of Danzig, 1532



Portrait of Georg Giese of Danzig, details

Figure 1.12: Hans Holbein the younger and his art.

Using anamorphic art to create “fake” architectural features, such as the domes below, became a sought-after signature skill of Andrea Pozzo.



Figure 1.13: These posters show how anamorphic art can be used to create fake spaces or to enhance architectural features.

Anamorphic street art is a popular modern art form. Some artists use their intuition alone, while others use mathematical principles to create their work.

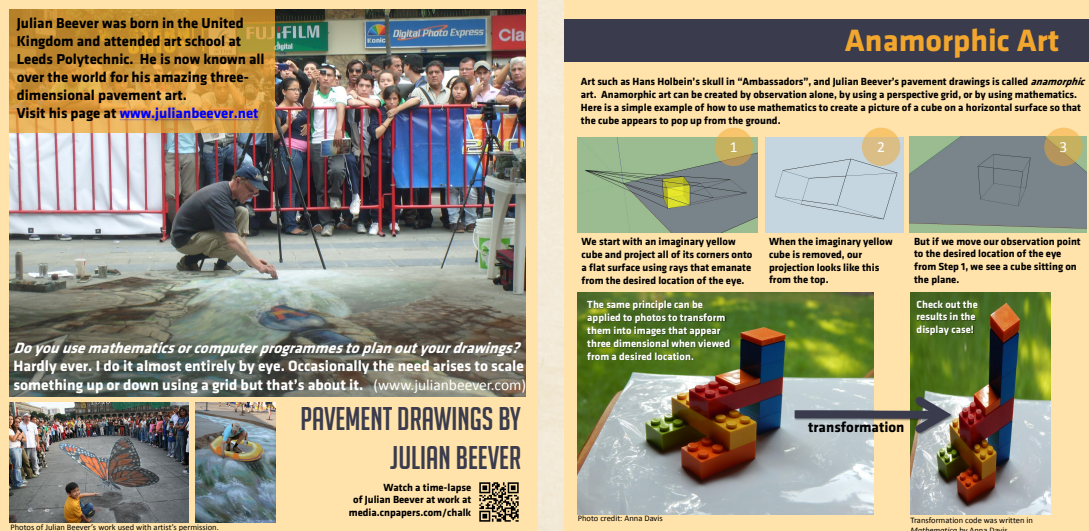


Figure 1.14: The poster on the left features the art of Julian Beever. The poster on the right features a photograph by Anna Davis distorted using an algorithm to create the display in Figure 1.15. The distortion algorithm was written by Anna Davis using Wolfram Mathematica.



Figure 1.15: A flat, distorted lego photo from Figure 1.14 is displayed on the left. The top portion of the photo had been cut away. When viewed from the correct vantage point, the legos in the flat photo appear three-dimensional (center photo). Young visitors were captured enjoying the anamorphic effect (right).



1	Points of Cubes	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
2	cube 1	x	y	z		new x	new y				line name	m	b		line name	m	b		vs	vb	2/c	
3		14.5625	64.4375	-8		15.97466	47.09291	1.0	1 T O 2	-1.87397	83.58778	5 T O 6	-1.78671	66.80381	0	14.5625	89.4375	-8				
4		28.0625	40.625	-14.375		33.70448	20.29549	2.0	1 T O 4	0.95690	28.45751				25	28.0625	70.625	-14.375				
5		42.75	50.1875	-21.5		49.70064	34.47011	3.0	1 T O 5	-3.62981	117.78307	6 T O 7	0.85141	-34.14626	0	42.75	84.1875	-21.5				
6		34.25	77.9375	-6.0625		36.61631	63.30442	4.0								34.25	102.9375	-6.0625				
7		12.4375	70.75	-31.0625		27.65873	17.38571	5.0	2 T O 3	0.89040	-9.77729					12.4375	95.75	-31.0625				
8		25.9375	52	-37.4375		38.35086	-1.56616	6.0	2 T O 6	-4.80748	184.60229					25.9375	77	-37.4375				
9		45.6875	65.5	-35.5		50.69335	-0.04778	7.0	3 T O 4	-2.17010	142.33139					45.6875	90.5	-35.5				
10		32.1875	84.3125	-29.125		40.70302	29.10512	8.0	3 T O 7	-25.64824	1109.20997					32.1875	109.3125	-29.125				
11	cube 2	48.25	29.9375	-3.0625		46.89728	18.43000	1.0	1 T O 2	0.210	-0.360	4 T O 5	7.776	-542.060	0	48.25	48.9375	-3.0625				
12		108.1875	36.375	-8.0625		102.94177	21.96456	2.0	1 T O 4	-0.787	87.085				25	108.1875	61.375	-8.0625				
13		94.9375	55.875	-12.4375		89.89761	31.79283	3.0	1 T O 7	4.151	-342.530	5 T O 7	-0.966	61.643	0	94.9375	80.875	-12.4375				
14		75	43.5	-7.5		73.60939	20.30531	4.0								75	68.5	-7.5				
15		73.25	37.0625	-30.5625		69.05631	-5.05243	5.0	2 T O 3	-0.726	96.664	6 T O 7	0.339	-41.430		73.25	62.0625	-30.5625				
16		106.4375	30	-31.125		91.55599	-10.37838	6.0	2 T O 6	2.841	-270.456					106.4375	55	-31.125				
17		80.5	17.5625	-26.125		78.80507	-14.64179	7.0								80.5	42.5625	-26.125				
18		93.1875	49.5	-35.5		81.50005	-1.38916	8.0	3 T O 4	0.159	17.607					93.1875	74.5	-35.5				
19	cube 3	39.625	95.875	-16.6875		43.73899	57.15193	1.0	1 T O 2	0.817	21.426	5 T O 6	0.502	-11.278	0	39.625	120.875	-16.6875				
20		61.625	91.9375	-8		61.44922	71.01993	2.0	1 T O 8	-6.574	344.690	5 T O 8	-0.513	45.122	25	61.625	116.9375	-8				
21		70.125	109.9375	-21.4375		67.64266	59.44675	3.0								70.125	134.9375	-21.4375				
22		48.125	118.875	-30.0625		51.84125	48.19519	4.0	2 T O 3	-1.966	192.402	6 T O 7	-0.937	88.077		48.125	138.875	-30.0625				
23		52.375	98.5	-48		55.50551	16.60026	5.0	2 T O 7	-15.801	1042.589					52.375	123.5	-48				
24		74.4375	94.5625	-39.375		69.04270	23.35587	6.0								74.4375	119.5625	-39.375				
25		65.875	76.5625	-25.9375		64.21754	27.87899	7.0	3 T O 6	-25.777	1803.091					65.875	101.5625	-25.9375				
26		43.875	80.5	-34.625		49.42360	19.76646	8.0								43.875	105.5	-34.625				
27	EYE	60	-96	66																		

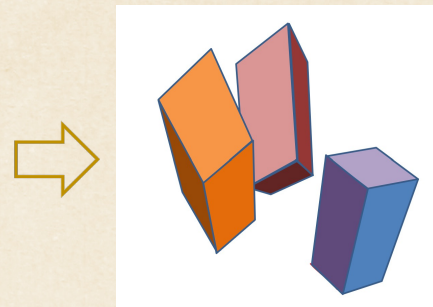


Figure 1.16: A large anamorphic centerpiece was installed in the middle of the gallery. This proved to be a logistical nightmare, as the cubes kept falling and setting off alarms in the middle of the night. The spreadsheet contains the actual calculations used to create the distorted image of the cubes.

STATION 5: EYE OF THE BEHOLDER

The camera obscura mimics the workings of the human eye and a photographic camera. This station explores the physics and psychology of sight.

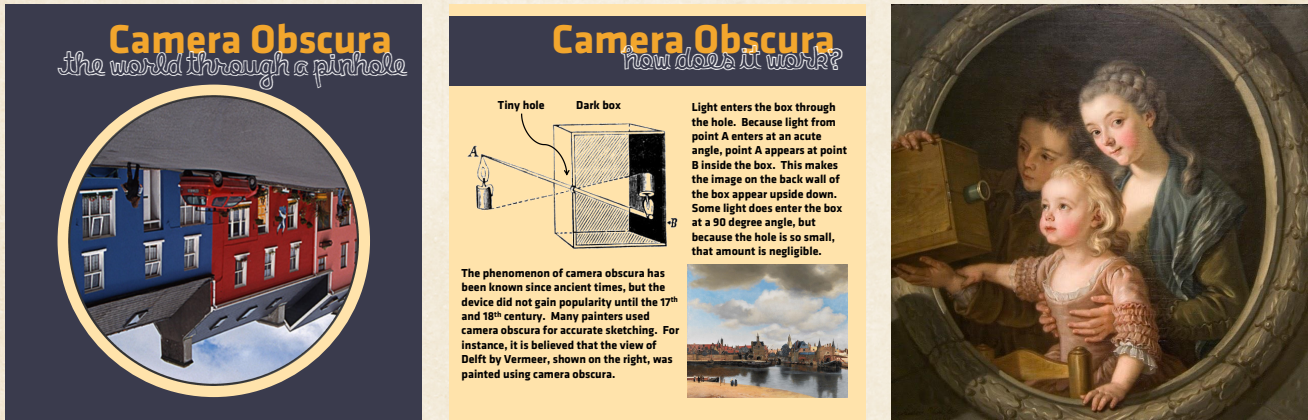


Figure 1.17: The physics and uses of camera obscura.



Figure 1.18: A working camera obscura, built by Col. Bill Snider and Anna Davis, allowed visitors to view cars drive by the gallery window (upside-down and backwards!) by putting their heads inside the box with a tiny pin-hole.

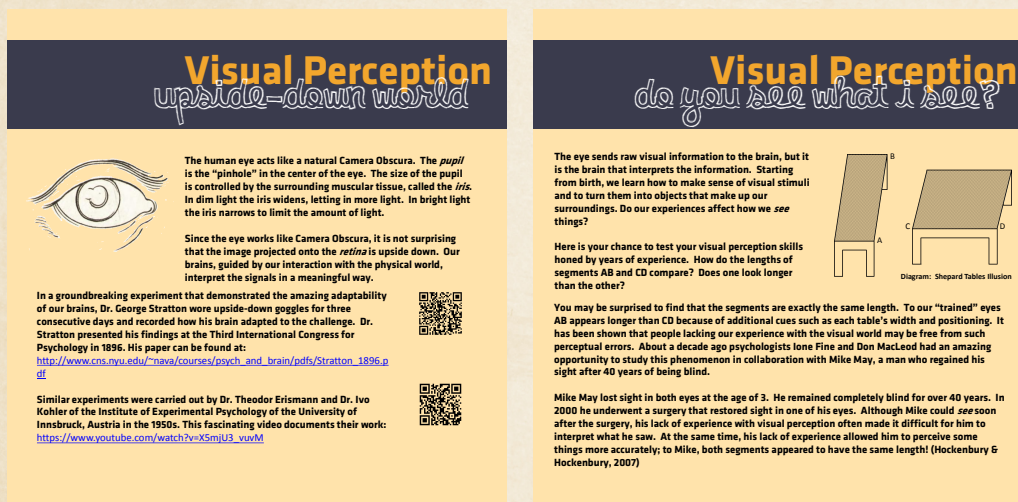


Figure 1.19: These posters discuss some interesting elements of the psychology of sight.

STATION 6: HIND SIGHT

What the audience learns from working with lines of sight can be easily applied to other linear phenomena. The following example describes an investigation performed by the *Time Scanners* team to solve a World War II mystery.



St. Paul's Cathedral

solving an old mystery



Photo Credit: Mark Fosh

St. Paul's Cathedral stands at the highest point in the city of London

St. Paul's Cathedral is an iconic fixture on the London skyline. St. Paul's as we know it today, was built to replace an earlier medieval structure which was destroyed in the Great Fire of London in 1666. The construction of the new Cathedral took place between 1675 and 1710 under the direction of Britain's most famous architect Sir Christopher Wren.



Sir Christopher Wren, 1632-1723



Photo Credit: Duff

Inside St. Paul's Cathedral



Amidst the devastation caused by the Blitz, St Paul's remained a beacon of hope.



Alison together watches the sky from a rooftop with St. Paul's in the background.

Christopher Wren was not trained as an architect but as a scientist and mathematician. It was his understanding of physics that allowed him to design and build one of the largest domes in northern Europe.

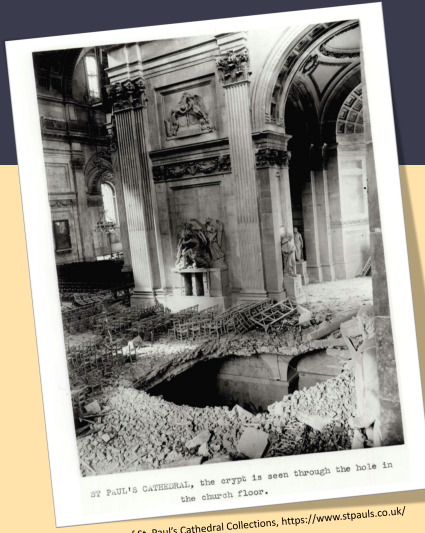
In 1940, the existence of St. Paul's was threatened by what became known as the Second Great Fire of London. As part of a campaign known as the Blitz, on December 29 and 30 the city of London was subjected to a massive airstrike.

The German Luftwaffe dropped firebombs and high explosives on the city for nearly twelve hours. Over twenty firebombs fell on the Cathedral and its grounds. As a massive fire raged all around, the Cathedral was saved from severe flame damage by a special volunteer firefighter brigade called the St. Paul's Watch.

Figure 1.20: St. Paul's Cathedral story.

St. Paul's Cathedral

where did the bomb explode?



ST PAUL'S CATHEDRAL, the crypt is seen through the hole in the church floor.

Photo courtesy of St. Paul's Cathedral Collections, <https://www.stpauls.co.uk/>

Firefighters could fight flames, but they were powerless against high explosive bombs. Such a bomb fell through the roof of the Cathedral only 25 feet away from the dome and exploded inside the structure. It is believed that had the bomb struck the dome, the Cathedral would have collapsed.

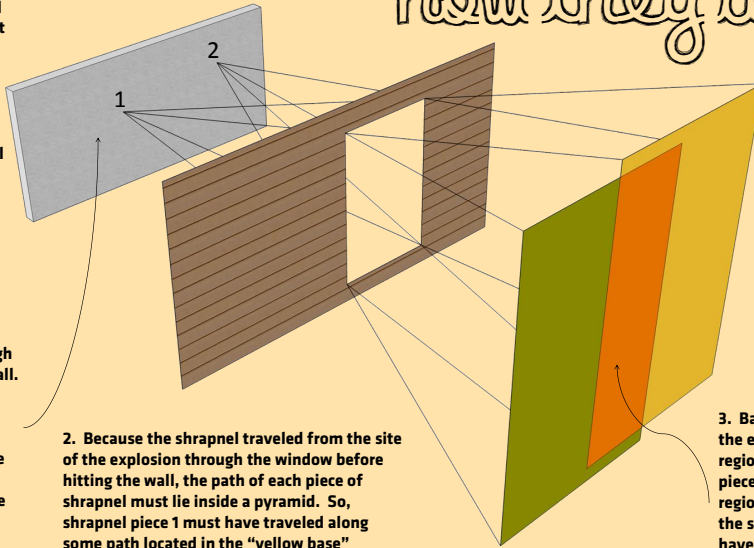
Because the explosion left a hole in the floor, it was assumed that the bomb fell through the floor and exploded in the crypt. A recent investigation by Steve Burrows and his team revealed that the hole in the floor was made by the powerful blast, not by the bomb falling through the floor. Using laser scanning technology and mathematical modeling investigators were able to pinpoint the exact location of the explosion that took place over 70 years ago! Here is how they did it.

Picture a fireworks display. Eventually all the sparks fall to the ground, but for a short period of time they disperse in all directions along seemingly straight lines. Similarly, when a bomb explodes, little pieces called shrapnel initially travel in all directions in straight lines. So, surprisingly, we can model the path of shrapnel using lines in the same way we model light rays.

1. When the bomb exploded, some shrapnel went out through the windows and hit a stone wall. Shrapnel damage inside the Cathedral has long been repaired, but many marks can still be seen on the wall outside the windows. This diagram shows two shrapnel marks. We will refer to them as 1 and 2.

2. Because the shrapnel traveled from the site of the explosion through the window before hitting the wall, the path of each piece of shrapnel must lie inside a pyramid. So, shrapnel piece 1 must have traveled along some path located in the "yellow base" pyramid while shrapnel piece 2 took a path inside of the "green base" pyramid.

how they did it:



5. Investigators were able to conclude that the blast occurred in mid-air, high above the floor. The hole in the floor was caused by the power of the blast rather than the bomb falling through the floor.

4. The blast left many shrapnel marks. As the investigators added more shrapnel data, the region where the explosion could have taken place shrank dramatically.

3. Based on the information from piece 1, the explosion occurred in the yellow region. Based on the information from piece 2, the explosion occurred in the green region. Because the two pieces came from the same explosion, the explosion must have happened where the yellow and the green region overlap.

Figure 1.21: Solution to the St. Paul's Cathedral mystery.

Just like knowing the location of the eye helps us construct accurate perspective renderings, working backwards from an accurate image can help us deduce the location of the eye. Working backwards from a photograph to figure out sizes and distances is sometimes called forensic photography. The following set of posters illustrates this topic.



Suzie loved taking walks with her Dad. Their walks were especially fun when they had a camera with them. One day Suzie and her dad took lots and lots of pictures by the railroad tracks. When they got home, Suzie discovered something amazing...

Look, Dad, look! Did you take this picture?

I don't know, Suzie. Maybe you did.

Can we figure out who did?

page 1

We sure can, Suzie.

But how?

Do you see the point where the rails appear to come together?

Yes, it's called the vanishing point.

Exactly! Well, the vanishing point is always located at the height of the eye.

But I'm not 3 inches tall!

page 2

You're right, Suzie, you're not 3 inches tall!

"We can print this photo in many different sizes," continued Dad.

"The vanishing point will appear at different heights, depending on the size of the print, but one thing will stay the same for all of the pictures."

"What will remain the same?" asked Suzie.

"Do you see the triangle made by the rails? Well, the triangle gets bigger or smaller with the picture, but the proportions of the triangle do not change."

So, the triangles made by the rails in each print are SIMILAR triangles!

page 3

Exactly, they are similar triangles!

Let me measure the base and the height of one triangle.

And I will record your measurements in the picture.

This tells us the ratio height/base

Prints may have different sizes, but the ratio of height to base will always be 7.2/11.8

Do we know the width of the actual tracks?

page 4

The standard railroad gauge in the US is 143.5 cm

"The ratio of the print triangle is 7.2/11.8," said Dad.

"The ratio of the real life triangle is ?/143.5," shouted Suzie.

That's very close to MY height, so I took that picture!!!

"So, in a life-size print, the vanishing point would be 87.6 cm high," explained Dad.

Great job, Suzie!

the end

Figure 1.22: A fairy tale that illustrates forensic photography.

No historical study of perspective would be complete without a mention of Filippo Brunelleschi. The following poster was located at the entry/exit to the exhibition.



FILIPPO BRUNELLESCHI
[1377-1446]

Filippo Brunelleschi is credited with the initial development of the theory of visual perspective. Brunelleschi was trained as a goldsmith and a sculptor, but gained fame as an architect and an engineer. His greatest achievement was the dome of Santa Maria del Fiore in Florence, Italy. Even today, the dome remains the largest brick dome in the world.

The Dome of Santa Maria del Fiore in Florence is known as the Duomo.
Photo Credit: Petar Milošević

Brunelleschi's interest in accurate perspective drawing arose from his desire to measure, sketch and study the buildings of Rome. According to the sixteenth century art historian Giorgio Vasari, "when he [Brunelleschi] came to Rome, and saw the grandeur of the buildings and the perfection of the form of the temples, he remained lost in thought and like one out of his mind; and he and Donatello set themselves to measure them and to draw out the plan of them, sparing neither time nor expense."
Upon returning to Florence, Brunelleschi made a painting of the Florentine Baptistery. To demonstrate the accuracy of his drawing technique, he drilled a hole in the back of the painting and had viewers look through the hole in the painting at the Baptistery while holding a mirror in front of them. The effect was that the painted building reflected in the mirror matched perfectly with the real building. Brunelleschi's contemporary, Antonio Manetti describes the experience as follows: "...the spectator felt he saw the actual scene when he looked at the painting. I have had it in my hands and seen it many times in my days, so I can testify to it."

 **View a Khan Academy video about Brunelleschi's experiment.**

Florentine Baptistery is the subject of Brunelleschi's mirror experiment.
Photo Credit: Lucrelli

Figure 1.23: Filippo Brunelleschi and his perspective experiment.

ACKNOWLEDGEMENTS

A very special thank you goes to Col. Bill Snider, Deb Tung and Harvey Tung for making very sturdy, aesthetically pleasing and functional props: the perspective frame, the wooden house, and the camera obscura.

I would also like to thank my son, Erik for suggestions, and Minecraft screenshots.

I am also grateful to the following organizations:

- Park National Bank for sponsoring the exhibition
- PCA Packaging Corporation of America for making the cardboard cubes
- Ohio Dominican University
- St. Paul's Cathedral Archives

CHAPTER 2: EDUCATIONAL IMPACT



VER one thousand school children from the surrounding school districts, and many members of the community attended the exhibition. Below are some of my favorite photos of visitors interacting with the exhibits during opening night and on school visits.



Figure 2.1: School field trip.

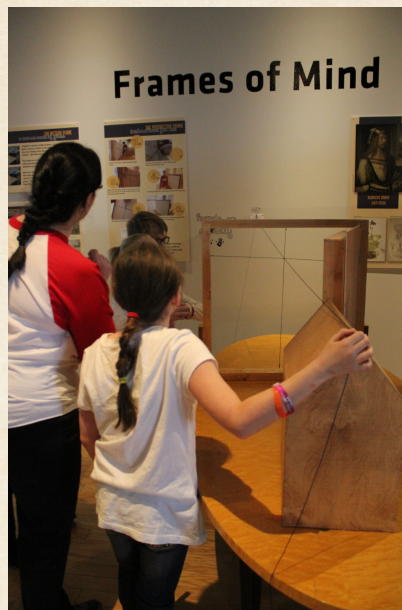


Figure 2.2: Students interacting with hands-on activities (left, center), and looking at fake domes (right).



Figure 2.3: Opening night.

KID TECH UNIVERSITY

Closing day featured a day camp for middle school students called *Kid Tech University*. Activities consisted of an interactive lecture on perspective, anamorphic art stations, and a video about St. Paul's Cathedral.

PERSPECTIVE LECTURE

Figure 2.4: The worksheet included activities that students completed during the lecture as well as QR codes to websites they can explore later.

ANAMORPHIC BUTTERFLY ACTIVITY STATION

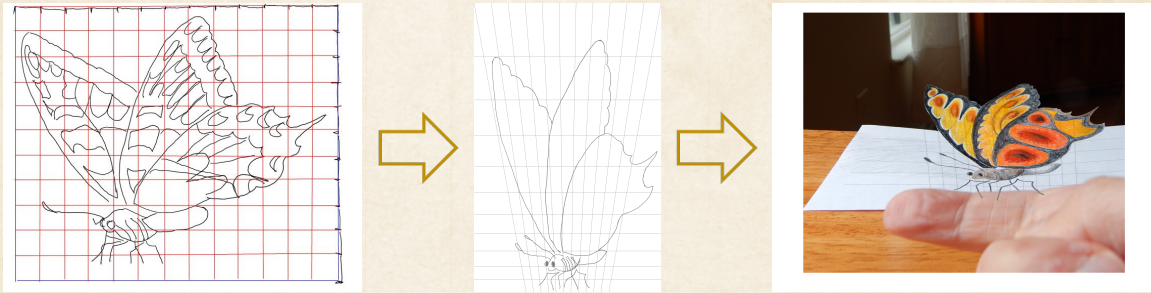


Figure 2.5: Metamorphosis of a butterfly: original sketch, sketch transformed using an anamorphic grid, anamorphic image is colored and cut out.



Figure 2.6: Butterfly created by a middle school student.

ANAMORPHIC CYLINDER ACTIVITY STATION

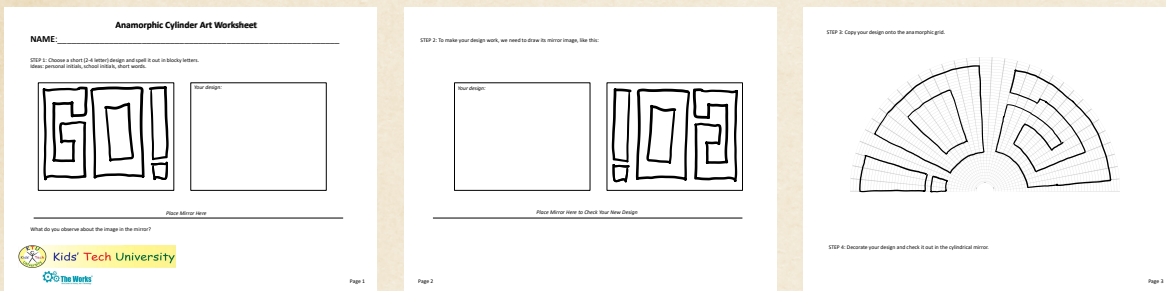


Figure 2.7: Students had an opportunity to create their own anamorphic art and to enjoy pre-made images.

OHIO STATE FAIR - JULY 2019

While hosting a booth at the *COSI Science Festival*, we were invited by the *Ohio Technology & Engineering Educators Association* to participate in a showcase at the *Ohio State Fair*. Having the showcase take place indoors allowed us to incorporate several digital displays into the show.

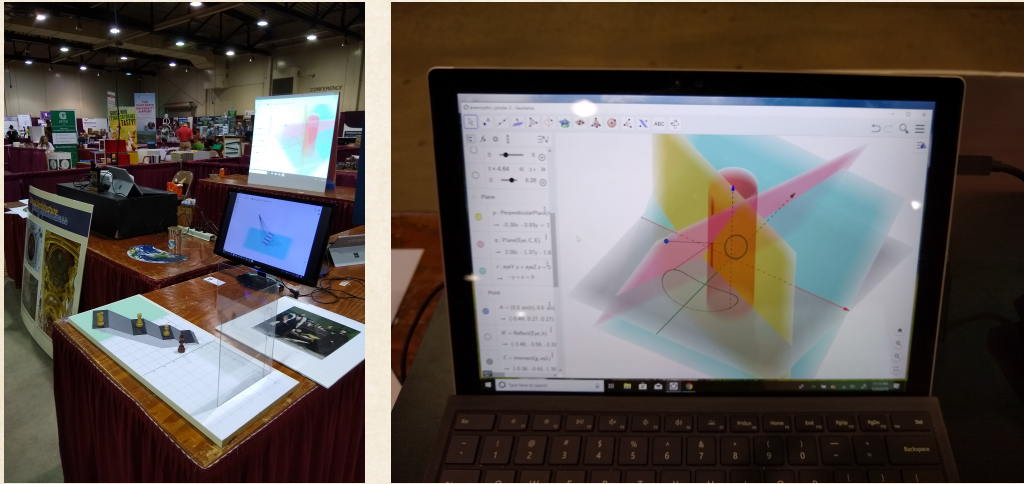


Figure 3.2: Digital animations explain the geometry behind the anamorphic staircase and the anamorphic cylinder.



Figure 3.3: We even got an award!

ACKNOWLEDGEMENTS

Many thanks to
Manuel Martinez, for organizing the *COSI* booth and hanging out with us in the rain.
Ron Zielke, for always being ready to take part in a new perspective adventure.
Ohio Technology and Engineering Educators Association, for inviting us to participate in the State Fair.

